Maximum Electric and Magnetic Field Strengths at a Given Distance from Some Ideal Antennas

Frédéric Broydé and Evelyne Clavelier

Abstract

This paper is about the computation of the maximum electric and magnetic field strengths at a given distance of an antenna, close to the antenna, the maximum being taken over all orientations. We provide closed-form expressions for the maximum field strengths produced in free space by three important ideal antennas: the electric Hertzian dipole, the magnetic Hertzian dipole, and the half-wave dipole.

Keywords: Dipole antennas; mobile antenna; near field; antenna theory; electromagnetic fields

1. Introduction

A portable wireless transmitter with an integral antenna is often regarded by electromagnetic compatibility (EMC) engineers as a emitter of electromagnetic disturbances capable of degrading the operation of a nearby apparatus, rather than a source of wanted electromagnetic radiation. This degradation is called interference, and the apparatus is referred to as the susceptible device (or victim). The immunity level of a susceptible device to such radiated disturbances is the maximum level of the incident electric or magnetic field for which the susceptible device remains capable of operating at a required degree of performance [1].

On the one hand, immunity standards define a minimum required immunity level (the immunity limit), as a function of frequency, for unmodulated or modulated disturbances. On the other hand, the terminal conformance specifications applicable to a given type of portable transmitters usually define the nominal maximum output power of the transmitters, not the characteristics of their antennas. As far as the immunity of a given nearby susceptible device to electromagnetic disturbances is concerned, the antenna is therefore unspecified, and, additionally, the orientation of the antenna with respect to the susceptible device is random.

Consequently, the assessment of the maximum disturbance level close to a transmitting antenna – irrespective of the orientation – is most relevant to EMC, since it should be the basis for setting immunity limits relating to the radiated disturbances produced by portable transmitters, and the corresponding protection distances. This paper is about the assessment of the maximum electric or magnetic field strength in the vicinity of a few simple ideal antennas fed by an ideal source, in free space, as a function of the distance, d, to the antenna, the maximum being taken over all orientations. We note that in the near field, this maximum is of course *not* the field strength in the directions of maximum far-field radiation. Also, the distance, d, to the antenna shall mean the distance between a point of observation and the outer boundary of the antenna, i.e., the shortest distance between the point of observation and a point on the outer boundary of the antenna.

This problem is interesting because it is amenable to closed-form expressions, and it is relevant to the understanding of the possible behaviors and physical limitations of the unperturbed fields of antennas. Surprisingly, such closed-form expressions do not seem to have been published.

We first review the formulas found in some IEC [International Electrotechnical Commission] standards on the immunity to electromagnetic fields (Section 2) and on the definition of field regions (Section 3). We then introduce (Section 4) four models of antennas. We derive (Sections 5 and 6) closed-form expressions for computing the maximum free-space electric and magnetic field strengths at a given distance from three ideal antennas: the electric Hertzian dipole, the magnetic Hertzian dipole, and the half-wave dipole. We then discuss (Section 7) possible applications of our results to EMC engineering.

2. Information Available in Standards

Information on the electromagnetic fields produced by a nearby antenna can be found in several basic international EMC standards. The IEC 61000-2-3 report [2] on the description of the electromagnetic environment essentially presented the classical free-space far-field formula:

$$E = \frac{k_1 \sqrt{W_{ERP}}}{r}, \qquad (1)$$

where W_{ERP} is the effective radiated power (ERP) in the direction of maximum far-field radiation, referred to a $\lambda/2$ dipole antenna [1]. This is equal to the product of the power delivered to the

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antenna by the maximum antenna gain, divided by 1.64 (i.e., the maximum gain of an ideal $\lambda/2$ dipole antenna). r is the distance between the point of observation and the reference point of the antenna (for instance, the center of a dipole antenna), assumed to be much larger than the wavelength, λ divided by 2π. E is the rms electric field strength, and k_1 is a coefficient equal to $7.0\,\Omega^{1/2}$.

Annex A of the 1984 edition of the IEC 801-3 standard [3] said that the "statistical average" of the field of a commercial portable transmitter could be expressed as

$$E = \frac{k_2\sqrt{W_{NOM}}}{r}, \qquad (2)$$

where W_{NOM} is the power claimed by the manufacturer of the commercial portable transmitter, k_2 is a coefficient characteristic of the portable transmitter, and the other quantities have the same meaning as previously. This standard gave a statistical average of $k_2 = 1.6\,\Omega^{1/2}$, based on measurements, without reference to published results. It also gave qualitative information on the near field, mentioning, for instance, the faster decrease of fields with distance.

Annex A of the first edition of the IEC 1000-4-3 standard [4] used Equation (2), but indicated a statistical average of $k_2 = 3.0\,\Omega^{1/2}$, apparently based on the same investigation as the one cited in 801-3. Information on the near field was removed.

Annex E of the third edition of the IEC 61000-4-3 standard [5] reintroduced Equation (1) when the ERP is known, and suggested the use of Equation (2), with a typical value of $k_2 = 3.0\,\Omega^{1/2}$ otherwise. There was still no quantitative information on the field strength occurring in the near field.

3. Field Regions

In the study of the immunity of a device, the engineer has a strong interest in the near field. However, first of all, what do we mean by the far and near fields? The far-field region is the region beyond the near-field region, where the relative angular distribution of fields no longer varies with distance. Of course, the transition from the near-field region is not sharply defined. The border between the near-field and far-field regions is usually considered to lie at a distance R_{FF} from the reference point of the antenna [6, §1-3; 7, p. 33-4; 8-10] equal to

$$R_{FF} = \max\left(\frac{2D^2}{\lambda}, \frac{\lambda}{2\pi}\right), \qquad (3)$$

where D is the maximum overall dimension of the antenna. This distance, R_{FF}, is the maximum of two terms. The first term corresponds to the outer limit of the radiating near-field region, and the second term corresponds to the outer limit of the reactive near-field region. In the case of a $\lambda/2$ dipole, we get $R_{FF} = \lambda/2$, instead of the value $\lambda/2\pi$ mentioned in Section 2. Up until now, simple formulas like Equation (1) only apply to the far field. This is because they have been established using a far-field approximation that requires $r \gg R_{FF}$, for instance $r > 12.4R_{FF}$ in the case of a half-wave dipole, according to [9].

4. Four Simple Antenna Models

In the far-field region, the electric field strength, E, and the magnetic field strength, H, are related by $E = \eta_0 H$, where $\eta_0 \approx 376.7\,\Omega$ is the intrinsic impedance of free space. This simple relationship does not apply in the near field. Also, at a given point, the knowledge of E and H are equally important, since a high-impedance electrically small circuit is mainly susceptible to the electric field, whereas a low-impedance electrically small circuit is mainly susceptible to the magnetic field. Consequently, any description of the near field must provide values for the electric field strength and the magnetic field strength.

We can also use the magnetic electric field strength [11, 12], defined as $E_M \equiv \eta_0 H$, in place of the magnetic field strength (EMC engineers have a habit of expressing all immunity limits in V/m, since they cannot say that an interference is caused by the electric field or the magnetic field, during immunity tests). In the far field, we have $E_M = E$, but this equality will not hold in the near field.

In this section, we consider the following ideal single-element wire antennas:

- The electrically small electric dipole (also called an electric Hertzian dipole),
- The electrically small magnetic dipole (also called a magnetic Hertzian dipole),
- The half-wave dipole with sinusoidal current distribution,
- The half-wave dipole with an almost-exact current distribution.

The field strengths produced by these antennas in the plane of maximum far-field radiation (i.e., the azimuth plane $\theta = \pi/2$) as a function of the distance, r, between the point of observation and the reference point of the antenna, are compared in Figure 1 [11, 12]. Again, let us stress that in the near field, Figure 1 does not show the maximum field strength. Also, r is not the distance d defined in Section 1 and used in the next figures, and in Sections 5 and 6.

Electric and magnetic Hertzian dipoles are studied in most textbooks [13, §8.4-§8.6]. These provide formulas for the electric and magnetic field components in free space as a function of r and zenith angle, θ, for a given dipole moment. Such formulas assume that $D \ll \lambda$ and $r \gg D$. In the near field, the field strengths shown in Figure 1 for the Hertzian dipoles increase rapidly when the distance is reduced. In the far field, they correspond to an antenna gain of 1.5.

The half-wave dipole with sinusoidal current distribution is a hypothetical antenna that produces a far-field field strength corresponding to a gain of 1.64. Analytical expressions are available for computing the fields at all distances from this antenna [13, §8.11; 14, §2.7] as a function of zenith angle, θ. In the near field, we note that the electric field strength levels off.

In this paper, the half-wave dipole with an almost-exact current distribution model is a numerical model. It is based on the implementation of the Method of Moments with point matching

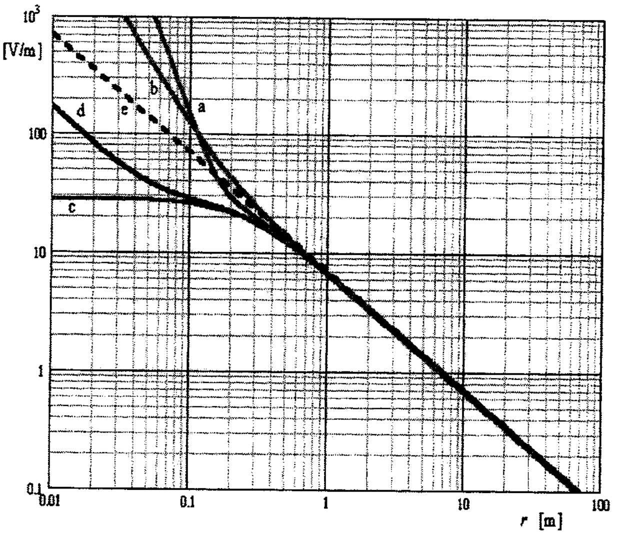

Figure 1. The rms field strengths produced by the four simple antenna models in the plane of maximum far-field radiation, as a function of the distance, *r*, to the reference point of the antenna, for $\lambda = 1\,\text{m}$ and a power $W = 1\,\text{W}$ delivered to the antenna. (a) is *E* for the electric Hertzian dipole and E_M for the magnetic Hertzian dipole; (b) is *E* for the magnetic Hertzian dipole and E_M for the electric Hertzian dipole; (c) is *E* for the $\lambda/2$ dipole with sinusoidal current distribution; (d) is *E* for the E_M dipole with almost-exact current distribution; (e) is E_M for both $\lambda/2$ dipole models.

for the computation of an "almost-exact" current distribution (in fact, it is, of course, an approximation). We have exactly followed the computational technique of §2.10 and §2.11 of [14], based on the Hallén integral equation. We did this for an antenna of total length $L = 0.48\lambda$, made of a cylindrical rod of diameter $0.01L$, using eight basis functions. The result was accurate at all distances, but the exact antenna characteristics have to be used in the model. This requires a significant computational time to obtain the maximum field strengths at a given distance *d* presented in Section 6. The far-field field strength corresponds to a gain of 1.64, like the $\lambda/2$ dipole with a sinusoidal current distribution. However, Figure 1 shows that the $\lambda/2$ dipole models behave differently in the reactive near-field region.

5. Field Strengths Produced by Hertzian Dipoles

We want to establish the maximum field strengths at a given distance, *d*, of a Hertzian dipole, regardless of θ. This is *not* the field strength for $\theta = \pi/2$. Since we always assume $D \ll d$ for Hertzian dipoles, the surfaces of constant distance *d* to the antenna (see Section 1) are spheres. The antenna is excited with a sinusoidal generator. Let us use *W* to denote the power radiated by the antenna, and *k* to denote the wavenumber, $2\pi/\lambda$. Then,

$$(kd)_C \equiv \sqrt{\frac{5+\sqrt{37}}{2}} \approx 2.354, \tag{4}$$

$$f = \begin{cases} 2\sqrt{(kd)^2+1} & \text{if } kd \le (kd)_C \\ \sqrt{(kd)^4-(kd)^2+1} & \text{if } kd > (kd)_C \end{cases}, \tag{5}$$

$$F_1 = \frac{1}{d}\sqrt{\frac{3\eta_0 W}{8\pi}}\frac{f}{(kd)^2}, \tag{6}$$

$$F_2 = \frac{1}{d}\sqrt{\frac{3\eta_0 W}{8\pi}}\frac{\sqrt{(kd)^2+1}}{kd}. \tag{7}$$

An elementary derivation shows that if we consider an electric Hertzian dipole, the maximum rms electric field strength is F_1 given by Equation (6), and the maximum rms magnetic electric field strength is F_2 given by Equation (7). The symmetry between the two types of Hertzian dipoles implies that if we consider a magnetic Hertzian dipole, the maximum rms electric field strength is F_2 given by Equation (7), and the maximum rms magnetic electric field strength is F_1 given by Equation (6). This result is shown in Figure 2. We observe that the curve F_2 of Figure 2 is close to the curve b of Figure 1, while in the near field the curve F_1 of Figure 2 is 6 dB above the curve a of Figure 1.

The maximum field strength given by the first line of Equation (5) and Equation (6) occurs on the straight line defined by the dipole moment, i.e., for $\theta = 0$. The maximum field strength given by the second line of Equation (5) and Equation (6), or by Equation (7), occurs in the plane orthogonal to the dipole moment, i.e., in the plane $\theta = \pi/2$.

6. Field Strengths Produced by $\lambda/2$ Dipoles

We are now interested by the maximum field strengths at a given distance from a half-wave dipole antenna, regardless of the zenith angle, θ. In this case, we observe that

- in the radiating near-field region, the maximum electric field strength occurs near the tips of the antenna, where θ is small;

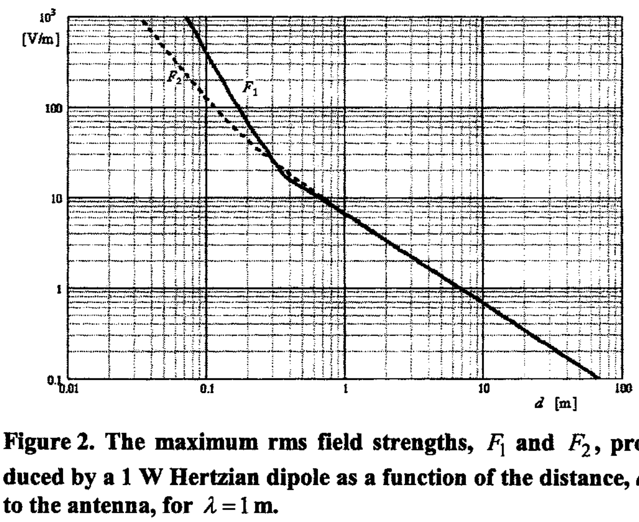

Figure 2. The maximum rms field strengths, F_1 and F_2, produced by a 1 W Hertzian dipole as a function of the distance, *d*, to the antenna, for $\lambda = 1\,\text{m}$.

- the maximum magnetic field strength always occurs in the plane of symmetry of the antenna, i.e., the plane $\theta = \pi/2$.

A surface of constant distance d to the antenna is made up of a portion of a cylinder and two hemispheres, as shown in Figure 3. Note that the distance d is not the distance r used in Section 2. Let us look for maximum field strengths on such surfaces, with d as a parameter. For a 300 MHz half-wave dipole antenna, we get the result shown in Figure 4 if we use the half-wave dipole with sinusoidal current distribution model, and the result shown in Figure 5 if we use the model of the half-wave dipole with almost-exact current distribution. We observe that the curves E of Figures 4 and 5 are completely different from the curves c and d of Figure 1, but are close to the curve e. Also, the curves E_M of Figures 4 and 5 are identical to the curve e of Figure 1.

For the computation of the almost-exact current distribution on a dipole antenna, we have only shown the results for a rod diameter equal to $L/100$, as noted in Section 4. A different rod diameter would produce similar characteristics, but the electric field strength would differ somewhat in the radiating near-field region. A thinner antenna would produce a curve closer to the curve applicable to the sinusoidal current distribution, because in

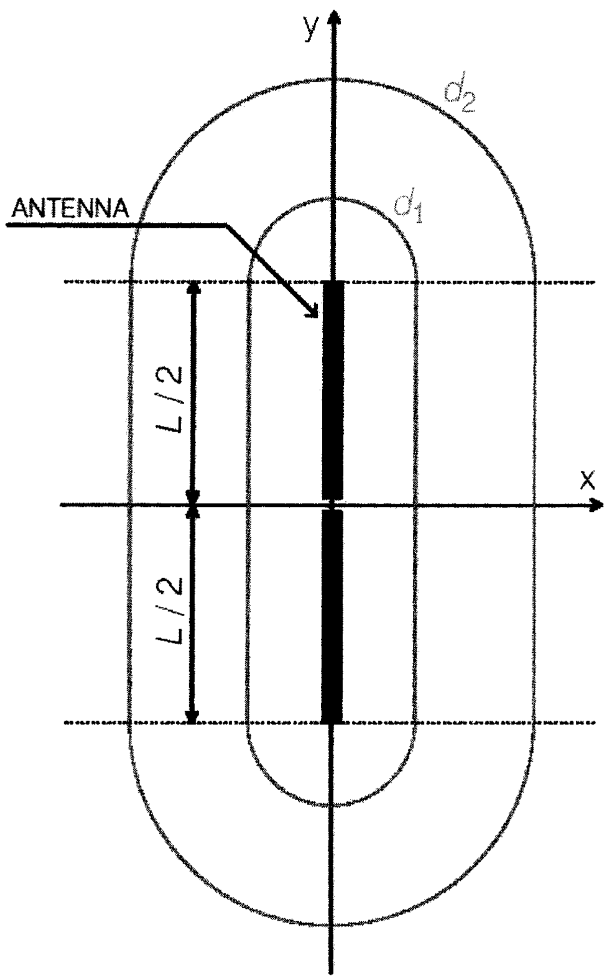

Figure 3. A dipole antenna and the intersection of the surfaces of constant distance to the antenna with a plane containing the antenna, for two distances, d_1 and d_2, to the antenna (that is to say, to the outer boundary of the antenna).

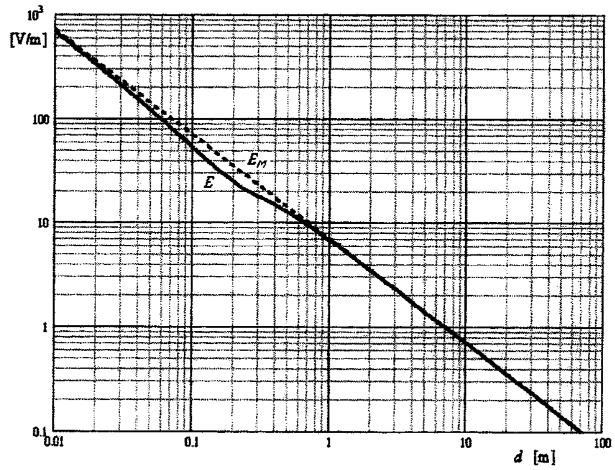

Figure 4. The maximum rms field strengths produced by a 1 W source as a function of the distance, d, to a $\lambda/2$ dipole, for $\lambda = 1$ m, using the model of the dipole with a sinusoidal current distribution.

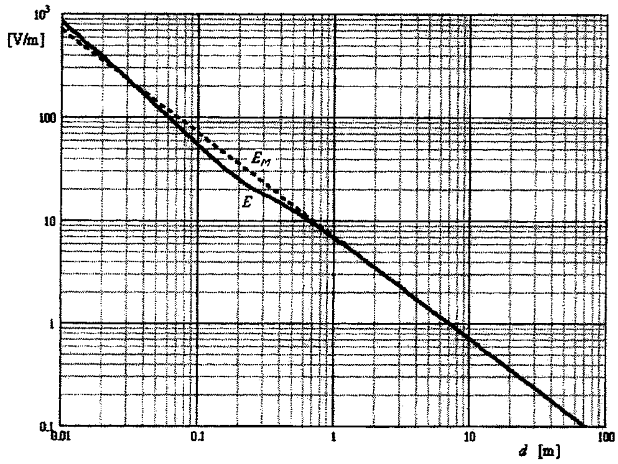

Figure 5. The maximum rms field strengths produced by a 1 W source as a function of the distance, d, to a $\lambda/2$ dipole, for $\lambda = 1$ m, according to the model of the dipole with almost-exact current distribution.

this case, the absolute value of the difference between the almost-exact current distribution and the sinusoidal current distribution decreases [15, Figure 8.13]. Changing the antenna's thickness would leave the magnetic field strength unchanged. The numerical computations leading to Figures 4 and 5 were performed using a standard set of *MATHCAD* worksheets [16].

Using W to denote the power radiated by the antenna, we found that the maximum rms field strengths produced by the half-wave dipole with sinusoidal current distribution may also be obtained using

$$E = \frac{\eta_0}{2\pi d}\sqrt{\frac{W}{R_0}}\,g\,, \tag{8}$$

where $R_0 \approx 73.13\,\Omega$, and

$$g = 1 - \frac{3.24\frac{d}{\lambda}}{\left(1+6.3\frac{d}{\lambda}\right)\left[1-3.5\frac{d}{\lambda}+17\left(\frac{d}{\lambda}\right)^2\right]}, \tag{9}$$

$$E_M = \frac{\eta_0}{2\pi d}\sqrt{\frac{W}{R_0}}. \tag{10}$$

Equation (8) is only a rational approximation of the results obtained numerically, but it is accurate to 0.1 dB. Using $g = 1$ instead of Equation (9) in Equation (8) introduces a maximum error of about 3 dB. Equation (10) is exact.

Figures 4 and 5 differ only for the electric field strength at very short distances from the antenna (for $d < 5\ \text{cm} \approx \lambda/20$), and by a small amount. Consequently, using the model of the half-wave dipole with sinusoidal current distribution may provide suitable accuracy down to very short distances from the antenna.

7. Discussion

In this communication, we considered antennas as emitters of electromagnetic disturbances in order to derive their EMC properties. We introduced the closed-form expressions of Equations (4) through (10), which give the maximum field strengths produced in free space by ideal Hertzian dipoles and by the half-wave dipole antenna with sinusoidal current distribution. They are applicable to all field regions. We have shown that Equations (8) through (10) can be used as a simplified model of an ideal half-wave dipole antenna with almost-exact current distribution, although it becomes somewhat inaccurate at very short distances.

These formulas assume lossless antennas. This assumption may be realistic for a real half-wave dipole, but in the case of Hertzian dipoles, significant losses are unavoidable in the radiating element and/or in the matching circuit. We may assume that losses in an antenna will affect the near field in the same manner as the far field. If W_{ERP} is known, this amounts to using Equations (4) through (7) with $W \approx 1.09 W_{ERP}$, since $1.64/1.5 \approx 1.09$, or to using Equations (8) through (10) with $W \approx W_{ERP}$, according to the case. It is worth noting that Equation (10) modified in this manner is Equation (1). For once, a not-so-straightforward theoretical problem (finding the maximum of the magnitude of a vector over a surface) leads to a simple analytical result!

EMC engineers might also be tempted to use Equations (4) through (10) as models for assessing the maximum field strengths produced by real antennas such that $D \geq \lambda/2$, regardless of their design. Since in the near field, the fields strongly depend on the surrounding items and on the antenna type and characteristics, such a quick estimate would clearly be a coarse approximation. However, this approximation would not conflict with the physics of antennas. For instance, Equations (8) through (10) have been applied to the terminal conformance specifications applicable to handsets used in Europe [17]. If this approximation is used, one should keep in mind that the so-called "monopole antennas" used in many portable transmitters behave as asymmetric dipoles [18], and that the planar antennas used as integral antennas in many modern UHF handsets suffer from a low efficiency (typically $\leq 50\%$), and might exhibit a behavior differing from our models [19].

In the future, it would be useful to develop analytical models for the assessment of the maximum field strength produced by antennas other than the simple antennas considered in this paper.

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