Some Properties of Multiple-Antenna-Port and Multiple-User-Port Antenna Tuners

Frédéric Broydé, *Senior Member, IEEE*, and Evelyne Clavelier, *Senior Member, IEEE*

***Abstract*—A user port impedance range and a user port tuning range characterize an antenna tuner at a given frequency. These sets and their local dimensions are defined and studied for an antenna tuner having any number n of antenna ports and any number m of user ports. A reciprocal antenna tuner is said to have a full tuning capability if it can provide or compensate all relevant small impedance variations, under the assumption that its adjustable impedance devices can provide suitable reactance values. For reciprocal antennas and a reciprocal antenna tuner, it is proven that the local dimension of the user port impedance range is equal to $m(m+1)$ if and only if the antenna tuner has a full tuning capability. We use this result to study an antenna tuner made of several uncoupled single-antenna-port and single-user-port antenna tuners. We also use this result to show that a new multiple-antenna-port and multiple-user-port antenna tuner has a full tuning capability.**

***Index Terms*—Antenna tuning, impedance matching, MIMO radio communication, radio receiver, radio transmitter.**

I. Introduction

AN ACTIVE equipment for radio communication, for instance a radio transmitter or a radio receiver, is referred to as the user, in this paper. A single-antenna-port and single-user-port (SAPSUP) antenna tuner is intended to be inserted between the user and its antenna, to be able to adjust the impedance seen by the user, so that it approximates a wanted impedance.

Fig. 1 shows a block diagram of a typical use of a SAPSUP antenna tuner, for tuning a single antenna which presents an impedance Z_{ant}. The antenna tuner comprises: an antenna port coupled to the antenna through a transmission line usually referred to as "feeder"; a user port (which may also be referred to as "radio port"); and one or more adjustable impedance devices. The antenna port sees an impedance Z_{Sant} and the user port presents an impedance Z_U. Each of the adjustable impedance devices has a reactance, this reactance being adjustable and having an influence on Z_U. Here, "adjustable impedance device" refers to any component having two terminals which behave as the terminals of a passive linear two-terminal circuit element, and which present an impedance which is adjustable by mechanical or electrical means.

A SAPSUP antenna tuner is the core of an automatic antenna tuner, also referred to as adaptive antenna tuner. Automatic SAPSUP antenna tuners may for instance be used in portable wireless devices, to provide the wanted impedance at different frequencies, and to compensate the so-called *user interaction*, also referred to as *user effect*, by which the body of a nearby person may severely degrade the performance of the antennas [1]–[10].

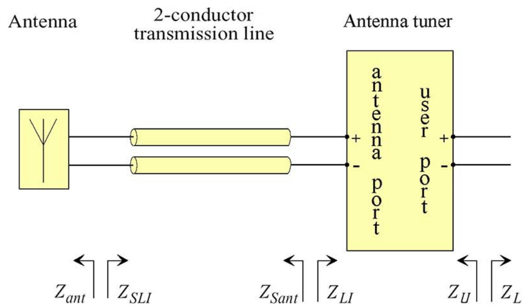

Fig. 1. An antenna connected to a 2-conductor transmission line connected to a SAPSUP antenna tuner.

A user may need to be coupled to several antennas, to use them simultaneously, in the same frequency band. This type of user may for instance be a receiver or a transmitter for MIMO radio communication. A multiple-antenna-port and multiple-user-port (MAPMUP) antenna tuner is intended to be inserted between such a user and an antenna array, to be able to adjust the impedance matrix presented by the user ports, so that it approximates a wanted impedance matrix. An antenna tuner must behave, with respect to the antenna ports and the user ports, as a passive linear device, in a frequency band of intended operation. In practice, losses are undesirable for signals applied to the antenna ports or the user ports, in the frequency band of intended operation. Thus, an ideal antenna tuner is lossless for signals applied to its antenna ports or user ports. The wanted impedance matrix is often a diagonal matrix, because in this case the user ports behave as the ports of uncoupled antennas having orthogonal radiation patterns, suitable for maximum power transfer or maximum capacity [11]–[15].

A MAPMUP antenna tuner may consist of uncoupled SAPSUP antenna tuners, one for each antenna [16]. The uncoupled antenna tuners need not be able to provide an impedance matrix presented by the user ports that approximates a wanted diagonal matrix. However, interesting results may be obtained with this kind of MAPMUP antenna tuner [17]–[22].

Recently, several MAPMUP antenna tuners which cannot be separated into uncoupled SAPSUP antenna tuners have been proposed [23]–[25].

This paper presents a theory of antenna tuners having any number of antenna ports and user ports, in which two sets play an important role: a user port impedance range and a user port tuning range. Two new quantities are defined in the paper: the local dimension (LD) of the user port impedance range and the LD of the user port tuning range. It is shown that a computation of the LD of the user port impedance range can be used to easily

Manuscript received June 05, 2014; revised August 22, 2014 and September 17, 2014; accepted October 07, 2014. Date of publication November 14, 2014; date of current version January 26, 2015. This paper was recommended by Associate Editor F. Bizzarri.

The authors are with Excem, 78580 Maule, France (e-mail: fredbroyde@eurexcem.com; eclavelier@eurexcem.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCSI.2014.2363513

determine whether the antenna tuner has a full tuning capability. This is our main result, and we explain how it can be used.

Section II states some known properties of a SAPSUP antenna tuner. The remainder of the paper consists of new results applicable to MAPMUP antenna tuners, which are related to the simpler aspects covered in Section II. Section III is used to define and investigate the user port impedance range and the user port tuning range, their LDs, and the full tuning capability. Section IV briefly covers the antenna port impedance range and the antenna port tuning range. In Sections V to VII, LDs are used to investigate the properties of several types of antenna tuners, including a novel MAPMUP antenna tuner having a full tuning capability.

II. Study of a SAPSUP Antenna Tuner

We use $\mathbb{R}$ to denote the set of real numbers and $\mathbb{C}$ to denote the set of complex numbers. $\mathbb{C}$ can be regarded as a real vector space of dimension 2, denoted by $E(1)$. Impedances are considered as functions of the frequency or the Laplace variable.

Let us consider a SAPSUP antenna tuner having p adjustable impedance devices, numbered from 1 to p. For any $j \in \{1,\ldots,p\}$, we use X_j to denote the reactance of the adjustable impedance device number j, at a specified frequency. At a given frequency, let us use f_U to denote the mapping such that

$$f_U(Z_{Sant}, X_1, \ldots, X_p) = Z_U \tag{1}$$

where each of the real variables $X_1, \ldots, X_p$ is any element of $\mathbb{R}$. An actual adjustable impedance device only provides, at a given frequency, reactance values lying either in a finite subset of $\mathbb{R}$, or in a bounded interval of $\mathbb{R}$. Thus, the capabilities of the adjustable impedance device number j define a set of achievable values of X_j. For instance, in current antenna tuner designs for cellular phones, an adjustable impedance device may consist of a barium strontium titanate (BST) varactor controlled by the output of a digital-to-analog converter, so that the set of achievable reactance values is finite [26], [27]. An adjustable impedance device made up of switched inductors, capacitors or stubs also provides a finite set of achievable reactance values [28], [29].

In (1), the impedance of an adjustable impedance device need not be a pure reactance, but (1) assumes that, at the given frequency, this impedance is a function of the reactance. This condition can be satisfied if any achievable value of X_j corresponds to a single setting of the adjustable impedance device number j.

The mapping f_U and the p sets of the achievable values of the real variables $X_1, \ldots, X_p$ can be used to define, at the given frequency, a user port impedance range, denoted by $D_{UR}(Z_{Sant})$, as the set of all $f_U(Z_{Sant}, X_1, \ldots, X_p)$ such that the value of each of the X_j is achievable. In other words, $D_{UR}(Z_{Sant})$ is the set of achievable values of Z_U, for a specified value of Z_{Sant}. It is a nonempty finite or infinite subset of $\mathbb{C}$.

As an example, we consider the antenna tuner shown in Fig. 2, intended to tune an ideal monopole antenna, having a total length of about 37.53 mm, coupled to a 54 mm-long feeder, so as to obtain that Z_U approximates a wanted impedance $Z_{UW} = 50\ \Omega$, at any frequency in a frequency band of center frequency $f_c = 806.5$ MHz, in spite of the user interaction. The $\lambda/4$ resonance of the antenna is near 1880 MHz. At f_c, the antenna and the feeder present the impedance $Z_{Sant} \approx (1.513 + j20.824)\ \Omega$. At f_c and for $L = 18$ nH, $Z_U = Z_{UW}$ is obtained for $C_U \approx 5.539$ pF and $C_A \approx 12.176$ pF. Fig. 3 shows $D_{UR}(Z_{Sant})$ of

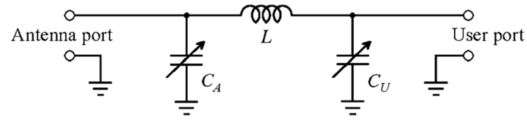

Fig. 2. A SAPSUP antenna tuner having a π-network structure.

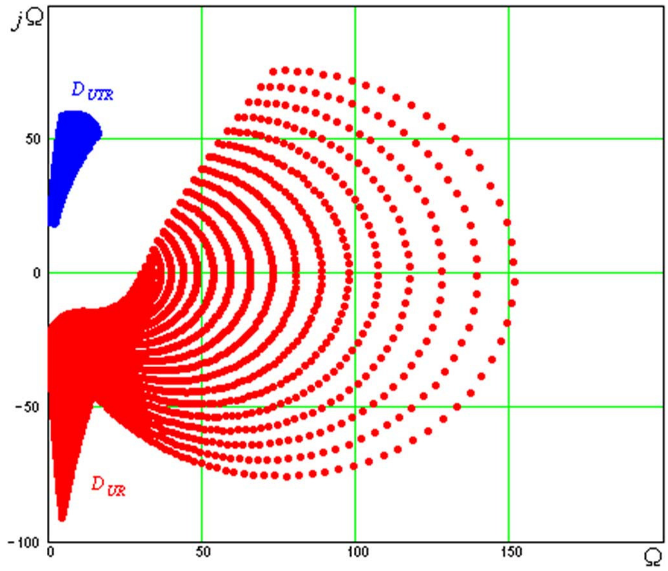

Fig. 3. The user port impedance range D_{UR} and the user port tuning range D_{UTR} of the SAPSUP antenna tuner.

this antenna tuner, at f_c, for $L = 18$ nH, C_U varying from 3.5 pF to 10 pF and C_A varying from 6.0 pF to 13 pF, both with 100 steps. Fig. 3 was obtained with a numerical analysis program, the method of moments with Lagrange polynomials as basis functions and point matching being used to compute the impedance of the antenna [30, ch. 2]. Though not inaesthetic, the shape of $D_{UR}(Z_{Sant})$ does not correspond to any named surface.

We can also define, at the given frequency, a user port tuning range, denoted by $D_{UTR}(Z_{UW})$, as the set of all Z_{Sant} for which there exist achievable values of the real variables $X_1, \ldots, X_p$ such that $f_U(Z_{Sant}, X_1, \ldots, X_p) = Z_{UW}$. Clearly, $D_{UTR}(Z_U)$ is a finite or infinite subset of $\mathbb{C}$, and it may be empty. Directly from the definitions, we obtain:

$$\{Z_{Sant} \in D_{UTR}(Z_U)\} \Leftrightarrow \{Z_U \in D_{UR}(Z_{Sant})\} \tag{2}$$

The user port tuning range may seem to be the most relevant parameter of the antenna tuner, because it shows the Z_{Sant} values which are compatible with the antenna tuner. For instance, in a configuration where the user is intended to operate with $Z_U = 50\ \Omega$, the antenna tuner is expected to provide a $D_{UTR}(50\ \Omega)$ which, in the frequency band of operation, contains impedances adequately spread over a neighborhood of the nominal value of Z_{Sant}. However, a manual or automatic adjustment of the antenna tuner is usually a closed-loop process based on measurements performed at the user port during emission, of Z_U or of the voltage standing-wave ratio [2], [6], [7], [10]. This process should find an optimum value of $\mathbf{Z}_U$, and it does not know anything about $\mathbf{Z}_{Sant}$. Thus, D_{UR} (Z_{Sant}) is important because it is what the operator or the automatic control system sees.

Fig. 3 shows $D_{UTR}(50\ \Omega)$ of the antenna tuner shown in Fig. 2, at f_c, for the component values and variations considered previously. The shape of $D_{UTR}(50\ \Omega)$ is a nameless surface,

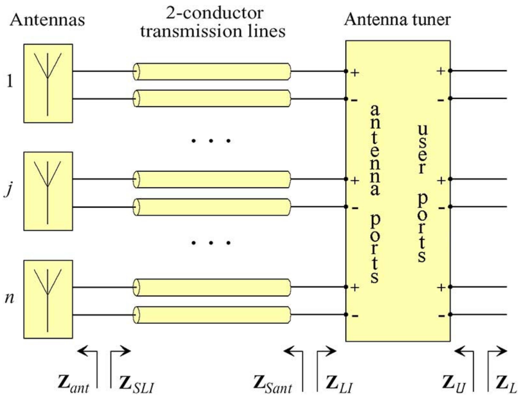

Fig. 4. An array of n antennas connected to n uncoupled 2-conductor transmission lines connected to a MAPMUP antenna tuner having n antenna ports and m user ports.

and it does not seem to be related to the shape of D_{UR} (Z_{Sant}) by a simple geometric transformation.

III. Considerations Based on the Impedance Matrix Presented by the User Ports

In this section, we extend the definitions of the user port impedance range and of the user port tuning range, to antenna tuners having any numbers of antenna ports and user ports. To cope with the fact that we are no longer able to plot and visualize these sets, we define and study their LDs, which can be used as figures of merit of the antenna tuner. We also establish their connection with a full tuning capability of the antenna tuner.

A. Definitions and Results Which Do Not Use Reciprocity

Let $\dim V$ denote the dimension of a vector space V over the field $\mathbb{R}$. Let $E(m)$, where m is a positive integer, be the vector space of complex matrices of size $m \times m$ over the field $\mathbb{R}$. We have $\dim E(m) = 2\,\mathrm{m}^2$. Impedance matrices are considered as functions of the frequency or of the Laplace variable.

Fig. 4 shows a block diagram of a typical use of a MAPMUP antenna tuner for tuning an array of $n \geq 1$ antennas which presents an impedance matrix $\mathbf{Z}_{ant}$. The antenna tuner comprises: n antenna ports each coupled to an antenna through a feeder, the antenna ports seeing an impedance matrix $\mathbf{Z}_{Sant}$ of size $n \times n$; $m \geq 1$ user ports presenting an impedance matrix $\mathbf{Z}_U$ of size $m \times m$; and p adjustable impedance devices. A user port may also be referred to as "radio port." Each adjustable impedance device has a reactance which is adjustable and has an influence on $\mathbf{Z}_U$. The impedance of an adjustable impedance device need not be a pure reactance, so that we are making no assumption on the antenna tuner.

Let us use $X_1, \ldots, X_p$ to denote the reactances of the adjustable impedance devices, as in Section II. At a given frequency, a mapping f_U may be defined by

$$f_U(\mathbf{Z}_{Sant}, X_1, \ldots, X_p) = \mathbf{Z}_U \qquad (3)$$

where each of the real variables $X_1, \ldots, X_p$ is any element of $\mathbb{R}$.

Using this mapping which fully characterizes the impedance matrix presented by the user ports, we may define, at the given frequency:

- A user port impedance range, denoted by $D_{UR}(\mathbf{Z}_{Sant})$, as the set of all $f_U(\mathbf{Z}_{Sant}, X_1, \ldots, X_p)$ such that the value of each of the X_j is achievable; and
- A user port tuning range, denoted by $D_{UTR}(\mathbf{Z}_{UW})$, as the set of all $\mathbf{Z}_{Sant}$ for which there exist achievable values of the real variables $X_1, \ldots, X_p$ such that f_U $(\mathbf{Z}_{Sant}, X_1, \ldots, X_p) = \mathbf{Z}_{UW}$.

For an arbitrary $\mathbf{Z}_{Sant}$, the set $D_{UR}(\mathbf{Z}_{Sant})$ is nonempty, but it follows from the definition that, for an arbitrary $\mathbf{Z}_{UW}$, the set $D_{UTR}(\mathbf{Z}_{UW})$ need not be nonempty. The definitions imply

$$\{\mathbf{Z}_{Sant} \in D_{UTR}(\mathbf{Z}_U)\} \Leftrightarrow \{\mathbf{Z}_U \in D_{UR}(\mathbf{Z}_{Sant})\} \qquad (4)$$

which generalizes (2). We see an important difference between the definitions of Section II and the present definitions: if $n \geq 2$ or $m \geq 2$, we are no longer able to visualize $D_{UR}(\mathbf{Z}_{Sant})$ and $D_{UTR}(\mathbf{Z}_U)$. This is why we are going to define and study a LD of $D_{UR}(\mathbf{Z}_{Sant})$ and a LD of $D_{UTR}(\mathbf{Z}_U)$.

$\mathbf{Z}_U$ may be considered as an element of $E(m)$. At given values of $X_1, \ldots, X_p$ and $\mathbf{Z}_{Sant}$, we define the tangent space of $D_{UR}(\mathbf{Z}_{Sant})$, denoted by T_U, as the span, in $E(m)$, of the partial derivatives $\partial f_U/\partial X_1, \ldots, \partial f_U/\partial X_p$.

Definition 1: The LD of $D_{UR}(\mathbf{Z}_{Sant})$ is $\dim T_U$. It is less than or equal to $\dim E(m) = 2\,\mathrm{m}^2$, and less than or equal to p. It is of course equal to the rank of $\partial f_U/\partial X_1, \ldots, \partial f_U/\partial X_p$ in $E(m)$.

Proposition 1: It follows directly from the definition that:

1) If the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is $2\,\mathrm{m}^2$, then $T_U = E(m)$, so that any small variation $\delta\mathbf{Z}_U$ in $\mathbf{Z}_U$ can be obtained, if suitable achievable values of $X_1, \ldots, X_p$ exist;
2) If the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is less than or equal to $2\,\mathrm{m}^2 - 1$, some of the conceivable small variations in $\mathbf{Z}_U$ cannot be obtained.

$\mathbf{Z}_{Sant}$ may be considered as an element of $E(n)$. At given values of $X_1, \ldots, X_p$ and $\mathbf{Z}_{Sant}$, let $\psi_E{:}E(n) \to E(m)$ be the partial differential of the function $f_U(\mathbf{Z}_{Sant}, X_1, \ldots, X_p)$ with respect to the variable $\mathbf{Z}_{Sant}$. This linear mapping is by definition such that, if $X_1, \ldots, X_p$ are kept constant, an arbitrary variation $\delta\mathbf{Z}_{Sant}$ in $\mathbf{Z}_{Sant}$ corresponds to a variation $\delta\mathbf{Z}_U$ in $\mathbf{Z}_U$, given by

$$\delta\mathbf{Z}_U = \psi_E(\delta\mathbf{Z}_{Sant}) + o\,(\|\delta\mathbf{Z}_{Sant}\|) \qquad (5)$$

to the first order in $\|\delta\mathbf{Z}_{Sant}\|$, where we have used Landau's "little o" notation. The partial differential ψ_E is defined without reference to any basis, but a choice of a basis allows us to express it using partial derivatives. We use $B_E = (\mathbf{E}_{E1}, \ldots, \mathbf{E}_{Eh})$, with $h = 2n^2$, to denote an orthonormal basis of $E(n)$, and we use $Z_{E1}, \ldots, Z_{Eh}$ to denote the coordinates of $\mathbf{Z}_{Sant}$ with respect to the basis B_E. For any $\mathbf{z} \in E(n)$, we have

$$\psi_E(\mathbf{z}) = \sum_{k=1}^{h} (\mathbf{E}_{E\,k} \cdot \mathbf{z}) \frac{\partial f_U}{\partial Z_{E\,k}} \qquad (6)$$

so that (5) may take on the equivalent form

$$\delta\mathbf{Z}_U = \sum_{k=1}^{h} (\mathbf{E}_{E\,k} \cdot \delta\mathbf{Z}_{Sant}) \frac{\partial f_U}{\partial Z_{E\,k}} + o\,(\|\delta\mathbf{Z}_{Sant}\|) \qquad (7)$$

which may look more familiar but cannot be used for the following considerations.

Proposition 2: There exist variations $\delta X_1, \ldots, \delta X_p$ in $X_1, \ldots, X_p$, respectively, which fully compensate, to the first order in $\|\delta\mathbf{Z}_{Sant}\|$, a small variation $\delta\mathbf{Z}_{Sant}$ in $\mathbf{Z}_{Sant}$, if and only if

$$\delta\mathbf{Z}_{Sant} \in T_{UT} \text{ where } T_{UT} = \psi_E^{-1}(T_U) \qquad (8)$$

where $\psi_E^{-1}(T_U)$ denotes the inverse image of T_U under ψ_E.

Proof: To obtain the wanted result, we need to solve

$$\sum_{k=1}^{p} \delta X_j \frac{\partial f_U}{\partial X_j} = -\psi_E(\delta \mathbf{Z}_{Sant}) \tag{9}$$

where the unknowns are the real variables $\delta X_1, \ldots, \delta X_p$. Since $\psi_E^{-1}(T_U)$ is always defined (even in the case where ψ_E is not an isomorphism and consequently not invertible), this equation has a solution if and only if (8) is satisfied. □

If ψ_E is an isomorphism, the equation $f_U(\mathbf{Z}_{Sant}, X_1, \ldots, X_p) = \mathbf{Z}_{UW}$ of unknown $\mathbf{Z}_{Sant}$ implicitly defines a differentiable mapping f_{UT} which satisfies $f_{UT}(\mathbf{Z}_{UW}, X_1, \ldots, X_p) = \mathbf{Z}_{Sant}$, so that $D_{UTR}(\mathbf{Z}_{UW})$ is the set of all $f_{UT}(\mathbf{Z}_{UW}, X_1, \ldots, X_p)$ such that the value of each of the X_j is achievable. By the definition of f_{UT}, we have

$$f_U\left(f_{UT}(\mathbf{Z}_{UW}, X_1, \ldots, X_p),\ X_1, \ldots, X_p\right) = \mathbf{Z}_{UW} \tag{10}$$

It follows that, for any $j \in \{1, \ldots, p\}$, we have

$$\sum_{k=1}^{h} \left(\mathbf{E}_{E\,k} \cdot \frac{\partial f_{UT}}{\partial X_j}\right) \frac{\partial f_U}{\partial Z_{E\,k}} = -\frac{\partial f_U}{\partial X_j} \tag{11}$$

so that

$$\frac{\partial f_{UT}}{\partial X_j} = -\psi_E^{-1}\left(\frac{\partial f_U}{\partial X_j}\right) \tag{12}$$

It follows that T_{UT} is the span of the partial derivatives $\partial f_{UT}/\partial X_1, \ldots, \partial f_{UT}/\partial X_p$ in $E(n)$. If ψ_E is not an isomorphism, it need not be possible to define the mapping f_{UT}.

B. Definitions and Results Using Reciprocity

Let $S(m)$ be the vector space of all symmetric complex matrices of size $m \times m$ over the field $\mathbb{R}$. $S(m)$ is a subspace of $E(m)$, and $\dim S(m) = m(m+1)$. We now assume that the antennas are reciprocal, so that $\mathbf{Z}_{ant}$ and $\mathbf{Z}_{Sant}$ lie in $S(n)$. We further assume that the antenna tuner is reciprocal, which in this paper means that the antenna tuner behaves, with respect to its antenna and user ports, as a reciprocal device. Thus, $\mathbf{Z}_U$ lies in $S(m)$, T_U is a subspace of $S(m)$, and the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is less than or equal to $m(m+1)$.

It follows from these assumptions that, for $m \geq 2$, we are only in the second case of Proposition 1. We also note that, though T_{UT} is a subspace of $E(n)$, it need not be a subspace of $S(n)$. For these reasons, specialized versions of Proposition 1 and Proposition 2 are needed, which only take into account the useful impedance matrix variations, that is to say the small symmetric variations in $\mathbf{Z}_U$ and the small symmetric variations in $\mathbf{Z}_{Sant}$.

Proposition 3: For reciprocal antennas and a reciprocal antenna tuner, it follows directly from the definition that:

1) If the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is $m\,(m+1)$, then $T_U = S(m)$, so that any small symmetric variation $\delta\mathbf{Z}_U$ in $\mathbf{Z}_U$ can be obtained, if suitable achievable values of $X_1, \ldots, X_p$ exist;
2) If the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is less than or equal to $m(m+1)-1$, some of the conceivable small symmetric variations in $\mathbf{Z}_U$ cannot be obtained.

Let $\psi_S : S(n) \to S(m)$ be the partial differential of the function $f_U(\mathbf{Z}_{Sant}, X_1, \ldots, X_p)$ with respect to the variable $\mathbf{Z}_{Sant}$. We use $B_S = (\mathbf{E}_{S1}, \ldots, \mathbf{E}_{Sg})$, with $g = n(n+1)$, to denote an orthonormal basis of $S(n)$, and we use $Z_{S1}, \ldots, Z_{Sg}$ to denote the coordinates of $\mathbf{Z}_{Sant}$ with respect to the basis B_S. The linear mappings ψ_S is such that, for any $\mathbf{z} \in S(n)$, we have

$$\psi_S(\mathbf{z}) = \sum_{k=1}^{g} (\mathbf{E}_{Sk} \cdot \mathbf{z}) \frac{\partial f_U}{\partial Z_{Sk}} \tag{13}$$

Proposition 4: For reciprocal antennas and a reciprocal antenna tuner, there exist variations $\delta X_1, \ldots, \delta X_p$ in $X_1, \ldots, X_p$, respectively, which fully compensate, to the first order in $\|\delta\mathbf{Z}_{Sant}\|$, a small symmetric variation $\delta\mathbf{Z}_{Sant}$ in $\mathbf{Z}_{Sant}$, if and only if

$$\delta\mathbf{Z}_{Sant} \in T_{UT} \cap S(n) = \psi_S^{-1}(T_U) \tag{14}$$

Proof: Since, for any $\mathbf{z} \in S(n)$, $\psi_E(\mathbf{z}) = \psi_S(\mathbf{z})$, we find that, for reciprocal antennas and antenna tuner and for a symmetric $\delta\mathbf{Z}_{Sant}$, (8) ⇔ (14). □

Definition 2: The LD of $D_{UTR}(\mathbf{Z}_U)$ is $\dim T_{UT} \cap S(n)$. It is less than or equal to $\dim S(n) = n(n+1)$. It is greater than or equal to the LD of $D_{UR}(\mathbf{Z}_{Sant})$.

Proposition 5: For reciprocal antennas and a reciprocal antenna tuner, it follows from Proposition 4 that:

1) If the LD of $D_{UTR}(\mathbf{Z}_U)$ is $n(n+1)$, then $T_{UT} = S(n)$, so that any small symmetric variation $\delta\mathbf{Z}_{Sant}$ in $\mathbf{Z}_{Sant}$ can be compensated to the first order in $\|\delta\mathbf{Z}_{Sant}\|$, if suitable achievable values of $X_1, \ldots, X_p$ exist;
2) If the LD of $D_{UTR}(\mathbf{Z}_U)$ is less than or equal to $n(n+1)-1$, some of the conceivable small symmetric variations in $\mathbf{Z}_{Sant}$ cannot be compensated to the first order in $\|\delta\mathbf{Z}_{Sant}\|$.

Proposition 6: For reciprocal antennas and a reciprocal antenna tuner, if the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is equal to $m(m+1)$, then the LD of $D_{UTR}(\mathbf{Z}_U)$ is equal to $n(n+1)$.

Proof: If the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is $m(m+1)$, then $T_U = S(m)$. Thus, $T_{UT} \cap S(n) = \psi_S^{-1}(T_U) = S(n)$. □

Proposition 7: For reciprocal antennas and a reciprocal antenna tuner, if the rank of the vectors $\partial f_U/\partial Z_{S1}, \ldots, \partial f_U/\partial Z_{Sg}$ is equal to $m(m+1)$, and if $m = n$, then the LD of $D_{UTR}(\mathbf{Z}_U)$ is equal to the LD of $D_{UR}(\mathbf{Z}_{Sant})$.

Proof: If the rank of the vectors $\partial f_U/\partial Z_{S1}, \ldots, \partial f_U/\partial Z_{Sg}$ is equal to $m(m+1)$ and $m = n$, then ψ_S is an isomorphism. Thus $\dim T_{UT} \cap S(n) = \dim \psi_S^{-1}(T_U) = \dim T_U$. □

C. Full Tuning Capability

We say that a reciprocal antenna tuner has a full tuning capability if it can provide or compensate all relevant small impedance variations, under the assumption that its adjustable impedance devices can provide suitable reactance values. A more rigorous definition follows.

Definition 3: For a given $\mathbf{Z}_{Sant}$ lying in $S(n)$, a reciprocal antenna tuner has a full tuning capability if:

- Any small symmetric variation $\delta\mathbf{Z}_U$ in $\mathbf{Z}_U$ can be obtained, if suitable achievable values of $X_1, \ldots, X_p$ exist; and
- Any small symmetric variation $\delta\mathbf{Z}_{Sant}$ in $\mathbf{Z}_{Sant}$ can be compensated to the first order in $\|\delta\mathbf{Z}_{Sant}\|$, if suitable achievable values of $X_1, \ldots, X_p$ exist.

Proposition 8: A reciprocal antenna tuner has a full tuning capability if and only if the LD of $D_{UR}(\mathbf{Z}_{Sant})$ equals the maximum possible value $m(m+1)$.

Proof: Direct consequence of propositions 3, 5, and 6. □

IV. Considerations Based on the Impedance Matrix Presented by the Antenna Ports

If the circuit connected to the user ports may be considered as linear, we can use $\mathbf{Z}_L$ to denote its impedance matrix, $\mathbf{Z}_{LI}$ to denote the impedance matrix presented by the antenna ports of the antenna tuner, and $\mathbf{Z}_{SLI}$ to denote the impedance matrix seen by the antennas, as shown in Fig. 4. In the case where antennas are used for reception, $\mathbf{Z}_{LI}$ determines the power transfer from the antennas and their feeders to the antenna tuner. It is therefore legitimate to investigate this impedance matrix. At a given frequency, a mapping f_A may be defined by

$$f_A(\mathbf{Z}_L, X_1, \ldots, X_p) = \mathbf{Z}_{LI} \tag{15}$$

where each of the real variables $X_1, \ldots, X_p$ is any element of $\mathbb{R}$.

Using this mapping which fully characterizes the impedance matrix presented by the antenna ports, we may define, at the given frequency:

- An antenna port impedance range, denoted by $D_{AR}(\mathbf{Z}_L)$, as the set of all $f_A(\mathbf{Z}_L, X_1, \ldots, X_p)$ such that the value of each of the X_j is achievable; and
- An antenna port tuning range, denoted by $D_{ATR}(\mathbf{Z}_{LIW})$, as the set of all $\mathbf{Z}_L$ for which there exist achievable values of the real variables $X_1, \ldots, X_p$ such that $f_A(\mathbf{Z}_L, X_1, \ldots, X_p) = \mathbf{Z}_{LIW}$.

For an arbitrary $\mathbf{Z}_L$, the set $D_{AR}(\mathbf{Z}_L)$ is nonempty, but it follows from the definition that, for an arbitrary $\mathbf{Z}_{LIW}$, the set $D_{ATR}(\mathbf{Z}_{LIW})$ need not be nonempty. The definitions imply

$$\{\mathbf{Z}_L \in D_{ATR}(\mathbf{Z}_{LI})\} \Leftrightarrow \{\mathbf{Z}_{LI} \in D_{AR}(\mathbf{Z}_L)\} \tag{16}$$

The reader can easily transpose the results of Section III on $D_{UR}(\mathbf{Z}_{Sant})$ and $D_{UTR}(\mathbf{Z}_U)$, to $D_{AR}(\mathbf{Z}_L)$ and $D_{ATR}(\mathbf{Z}_{LI})$. This provides the definition of the LD of $D_{AR}(\mathbf{Z}_L)$, and the LD of $D_{ATR}(\mathbf{Z}_{LI})$, and propositions relating to these LDs.

If the antenna tuner behaves as a lossless device with respect to its antenna ports and user ports, we can apply a known result on bilateral hermitian match [13, § III], [31, App.]. It follows that hermitian matching at the antenna ports entails hermitian matching at the user ports, and hermitian matching at the user ports entails hermitian matching at the antenna ports. Consequently, in this case we have

$$\{\mathbf{Z}_L^* \in D_{UR}(\mathbf{Z}_{Sant})\} \Leftrightarrow \{\mathbf{Z}_{Sant}^* \in D_{AR}(\mathbf{Z}_L)\} \tag{17}$$

where the star denotes the hermitian adjoint. This result can be combined with (4) and (16) to obtain

$$\{\mathbf{Z}_{Sant}^* \in D_{AR}(\mathbf{Z}_U^*)\} \Leftrightarrow \{\mathbf{Z}_{Sant} \in D_{UTR}(\mathbf{Z}_U)\} \tag{18}$$

and

$$\{\mathbf{Z}_L^* \in D_{ATR}(\mathbf{Z}_{Sant}^*)\} \Leftrightarrow \{\mathbf{Z}_L \in D_{UR}(\mathbf{Z}_{Sant})\} \tag{19}$$

Thus, for a lossless antenna tuner, the antenna port impedance range and the antenna port tuning range can be derived from the user port impedance range and the user port tuning range. This can be visualized in the case of a SAPSUP antenna tuner. For instance, Fig. 5 shows a direct computation of $D_{AR}(Z_L^*)$ and $D_{ATR}(Z_{Sant}^*)$ of the SAPSUP antenna tuner of Section II, for $Z_L^* = Z_L = 50\ \Omega$ and $Z_{Sant} \approx (1.513 + j20.824)\ \Omega$ at $f_c = 806.5$ MHz. A comparison with Fig. 3 shows that the symmetry expressed by (18) and (19) is indeed present.

Proposition 9: For a lossless and reciprocal antenna tuner and for $\mathbf{Z}_U$ and $\mathbf{Z}_{Sant}$ lying in $S(n)$, the LD of $D_{AR}(\mathbf{Z}_U^*)$ equals the LD of $D_{UTR}(\mathbf{Z}_U)$ and the LD of $D_{ATR}(\mathbf{Z}_{Sant}^*)$ equals the LD of $D_{UR}(\mathbf{Z}_{Sant})$.

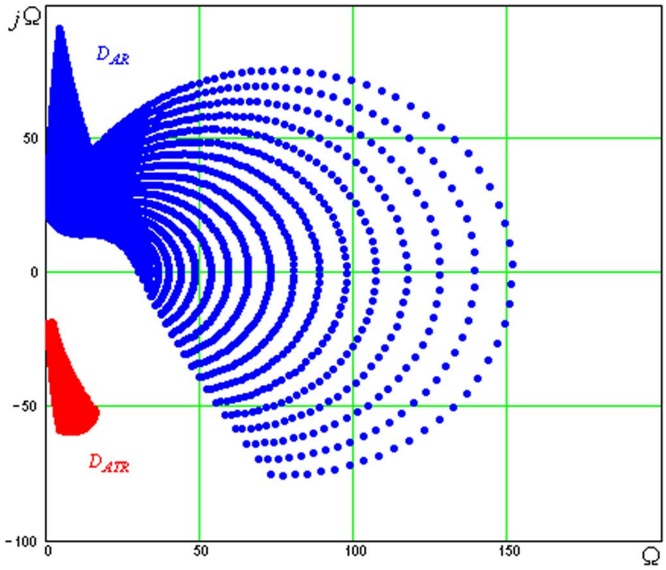

Fig. 5. The antenna port impedance range D_{AR} and the antenna port tuning range D_{ATR} of the SAPSUP antenna tuner.

Proof: The result can be derived from (18) and (19). □

Though (18) and (19) assume a lossless tuner, it may reasonably be assumed that Proposition 9 remains valid for antenna tuners having small losses.

V. Two Examples Using SAPSUP Antenna Tuners

Let us first consider the case of the SAPSUP antenna tuner used as an example in Section II, for $Z_{Sant} \approx (1.513 + j20.824)\ \Omega$ and at $f_c = 806.5$ MHz. For $C_U \approx 5.539$ pF and $C_A \approx 12.176$ pF, which corresponds to $Z_{UW} = 50\ \Omega$, but not exactly to achievable values of C_U and C_A, the LD of $D_{UR}(Z_{Sant})$ is 2. By Proposition 6, the LD of $D_{UTR}(50\ \Omega)$ is 2. Based on Proposition 8, it is possible to state that the SAPSUP antenna tuner has a full tuning capability for the given $\mathbf{Z}_{Sant}$.

This information is useful, but is not as rich as a plot of $D_{UR}(Z_{Sant})$ and $D_{UTR}(50\ \Omega)$ such as Fig. 3, which is not limited to small variations and which takes into account the achievable values of $X_1, \ldots, X_p$.

Let us now consider the case of a reciprocal MAPMUP antenna tuner made up of n uncoupled SAPSUP antenna tuners, as shown in Fig. 6. In the special case where $\mathbf{Z}_{Sant}$ is a diagonal matrix, $\mathbf{Z}_U$ is also a diagonal matrix because there is no coupling between the user ports. Consequently, the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is less than or equal to $2n = 2m$. In this case, Proposition 3 allows us to conclude that, for $n \geq 2$, some of the conceivable small symmetric variations in $\mathbf{Z}_U$ cannot be obtained. This result is not very useful since we are mostly interested in the configuration where $\mathbf{Z}_{Sant}$ is not a diagonal matrix, because the antennas interact.

Instead of assuming that $\mathbf{Z}_{Sant}$ is diagonal, let us now only assume reciprocal antennas. If we further assume that each SAPSUP antenna tuner is lossless and reciprocal, it is fully characterized by 3 real frequency dependent parameters (for instance: a reactance of the antenna port for an open-circuited user port, a reactance of the user port for an open-circuited antenna port and a transfer reactance). Thus, for a given $\mathbf{Z}_{Sant}$ and at a given frequency, $\mathbf{Z}_U$ is a function of at most $3n = 3m$

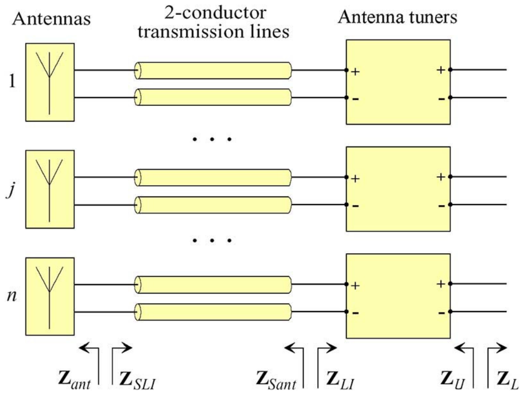

Fig. 6. An array of n antennas connected to n uncoupled 2-conductor transmission lines connected to n SAPSUP antenna tuners.

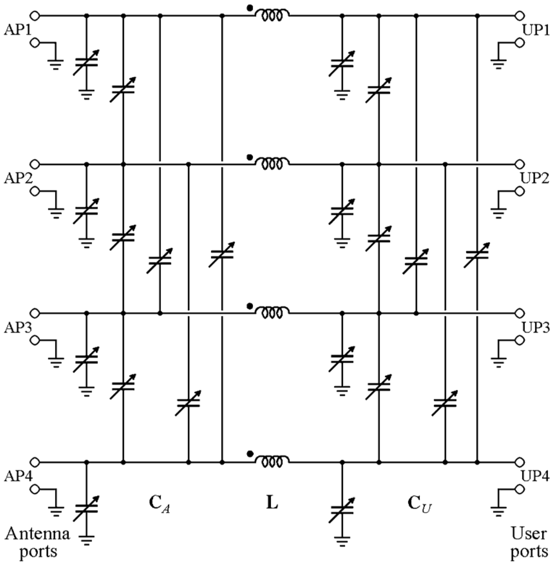

Fig. 7. A MAPMUP antenna tuner having $n = 4$ antenna ports, AP1 to AP4, and $m = 4$ user ports, UP1 to UP4.

independent real parameters. Thus, the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is less than or equal to $3m$. For $n \geq 3$, we have $3m < m(m+1)$, so that by Proposition 8, we conclude that the MAPMUP antenna tuner does not have a full tuning capability.

This is why, as said in the introduction, the uncoupled antenna tuners shown in Fig. 6 usually cannot be tuned to obtain a $\mathbf{Z}_U$ that is close to a wanted diagonal impedance matrix.

VI. Example of a New MAPMUP Antenna Tuner

We have searched antenna tuner structures which overcome the limitation explained in the previous section, of a MAPMUP antenna tuner made of uncoupled SAPSUP antenna tuners as shown in Fig. 6. This led to a MAPMUP antenna tuner having, in the special case $n = m = 4$, the schematic diagram shown in Fig. 7. Since it generalizes the antenna tuner circuit of Fig. 2 having a π-network structure, we say that it has the structure of a multidimensional π-network. This type of antenna tuner has $n(n+1) = m(m+1)$ adjustable impedance devices presenting a negative reactance, represented as variable capacitors in Fig. 7. Thus, there is a possibility that the adjustable impedance devices allow an independent control of the $m\ (m+1)$ real parameters which define the symmetric matrix $\mathbf{Z}_U$.

Let us assume that all adjustable impedance devices and all windings are lossless. We use $\mathbf{C}_A$ to denote the capacitance matrix of the $m(m+1)/2$ adjustable impedance devices coupled to one of the antenna ports, $\mathbf{L}$ to denote the inductance matrix of the windings and $\mathbf{C}_U$ to denote the capacitance matrix of the $m\ (m+1)/2$ adjustable impedance devices coupled to one of the user ports. Let ω be the radian frequency. We have

$$\mathbf{Z}_U = \left[\left[\left[\mathbf{Z}_{Sant}^{-1} + j\omega\mathbf{C}_A\right]^{-1} + j\omega\mathbf{L}\right]^{-1} + j\omega\mathbf{C}_U\right]^{-1} \quad (20)$$

We want to solve the problem of finding $\mathbf{C}_A$, $\mathbf{L}$, and $\mathbf{C}_U$ such that $\mathbf{Z}_U$ is equal to a wanted impedance matrix $\mathbf{Z}_{UW} = r_0\ \mathbf{1}_n$, where r_0 is a resistance and where, for a positive integer q, we use $\mathbf{1}_q$ to denote the identity matrix of size $q \times q$. The equation to be solved is

$$(g_0\mathbf{1}_m - j\omega\mathbf{C}_U)^{-1} = \left(\mathbf{Z}_{Sant}^{-1} + j\omega\mathbf{C}_A\right)^{-1} + j\omega\mathbf{L} \quad (21)$$

where $g_0 = 1/r_0$. Let us introduce the real matrices $\mathbf{G}_A$ and $\mathbf{B}_A$ which satisfy $\mathbf{Z}_{Sant}^{-1} = \mathbf{G}_A + j\mathbf{B}_A$. If $\mathbf{Z}_{Sant}$ is symmetric, the design of the MAPMUP antenna tuner providing $\mathbf{Z}_U = \mathbf{Z}_{UW}$ in a given frequency band can use the following steps:

Step 1) Select an arbitrary physically realizable capacitance matrix $\mathbf{C}_A$ and an arbitrary frequency f_A in the frequency band;

Step 2) Compute $\mathbf{C}_U$ compatible with $\mathbf{Z}_U = \mathbf{Z}_{UW}$ at f_A, using

$$\omega\mathbf{C}_U = \left[g_0\mathbf{G}_A + g_0(\mathbf{B}_A + \omega\mathbf{C}_A)\mathbf{G}_A^{-1}(\mathbf{B}_A + \omega\mathbf{C}_A) - g_0^2\mathbf{1}_m\right]^{\frac{1}{2}} \quad (22)$$

Step 3) Compute $\mathbf{L}$ providing $\mathbf{Z}_U = \mathbf{Z}_{UW}$ at f_A, using

$$\omega\mathbf{L} = \left[g_0^2\mathbf{1}_n + (\omega\mathbf{C}_U)^2\right]^{-1}\omega\mathbf{C}_U + \left[\mathbf{B}_A + \omega\mathbf{C}_A + \mathbf{G}_A(\mathbf{B}_A + \omega\mathbf{C}_A)^{-1}\mathbf{G}_A\right]^{-1} \quad (23)$$

Step 4) Determine whether $\mathbf{C}_U$ and $\mathbf{L}$ are realizable; if no, go back to step 1; if yes, a physically realizable solution of $\mathbf{Z}_U = \mathbf{Z}_{UW}$ at f_A has been obtained.

The voltage transfer matrix from the user port to the antenna port, denoted by $\mathbf{T}_U$, is given by

$$\begin{aligned}\mathbf{T}_U &= \left(\mathbf{Z}_{Sant}^{-1} + j\omega\mathbf{C}_A\right)^{-1}\left[\left(\mathbf{Z}_{Sant}^{-1} + j\omega\mathbf{C}_A\right)^{-1} + j\omega\mathbf{L}\right]^{-1} \\ &= \left[\mathbf{1}_m + j\omega\mathbf{L}\left(\mathbf{Z}_{Sant}^{-1} + j\omega\mathbf{C}_A\right)\right]^{-1}\end{aligned} \quad (24)$$

According to [32]–[34], for a lossless network, $\mathbf{T}_U$ satisfies

$$\mathbf{T}_U = \sqrt{g_0}\mathrm{Ch}(\mathbf{G}_A)^{-1}\mathbf{U} \quad (25)$$

where $\mathrm{Ch}(\mathbf{G}_A)$ denotes the result of the Cholesky decomposition of the positive definite matrix $\mathbf{G}_A$, and $\mathbf{U}$ is an arbitrary unitary matrix. This allows us to interpret step 1: different choices of $\mathbf{C}_A$ lead to different values of $\mathbf{T}_U$ which correspond to different values of $\mathbf{U}$. In the case $n = m = 1$ used in the example of Section II, different values of C_A correspond to a change in the phase of the voltage transfer ratio and to a change in other electrical characteristics of the antenna tuner, such as the loaded quality factor [1], [35]. However, different values of C_A have no impact on the directivity pattern. In the case $n = m \geq 2$ of Fig. 7, different choices of $\mathbf{C}_A$ may change the electrical characteristics of the antenna tuner and the directivity pattern of the user ports. In a portable wireless device such as a mobile phone, for which the orientation is random and time-varying, and for which the directivity pattern is subject to the user interaction, we believe that this is not a problem.

As an example, let us assume that the antenna tuner is intended to tune an antenna array operating in the frequency band 1850 MHz to 1910 MHz. The antenna array is a circular array

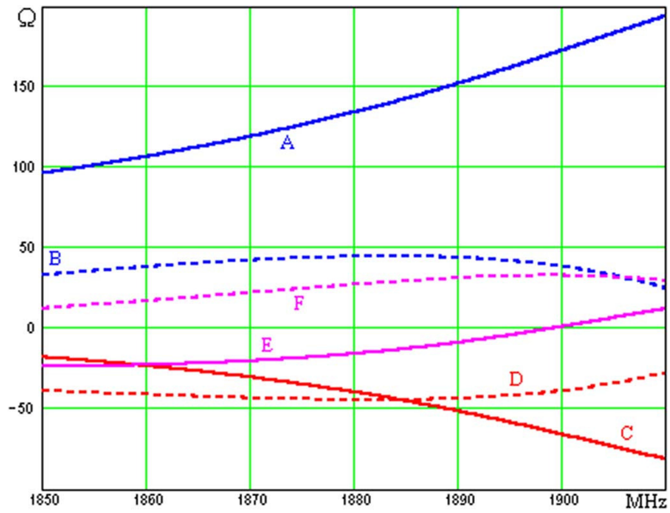

Fig. 8. Entries of $\mathbf{Z}_{Sant}$ versus frequency: $\mathrm{Re}(\mathbf{Z}_{Sant\ 11})$ is curve A; $\mathrm{Im}(\mathbf{Z}_{Sant\ 11})$ is curve B; $\mathrm{Re}(\mathbf{Z}_{Sant\ 12})$ is curve C; $\mathrm{Im}(\mathbf{Z}_{Sant\ 12})$ is curve D; $\mathrm{Re}(\mathbf{Z}_{Sant\ 13})$ is curve E; $\mathrm{Im}(\mathbf{Z}_{Sant\ 13})$ is curve F.

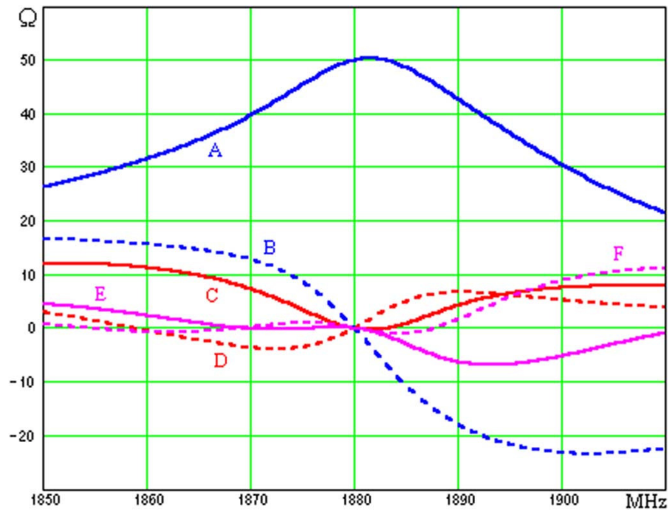

Fig. 9. $\mathbf{Z}_U$ for the initial values of $\mathbf{C}_A$ and $\mathbf{C}_U$: $\mathrm{Re}(\mathbf{Z}_{U\ 11})$ is curve A; $\mathrm{Im}(\mathbf{Z}_{U\ 11})$ is curve B; $\mathrm{Re}(\mathbf{Z}_{U\ 12})$ is curve C; $\mathrm{Im}(\mathbf{Z}_{U\ 12})$ is curve D; $\mathrm{Re}(\mathbf{Z}_{U\ 13})$ is curve E; and $\mathrm{Im}(\mathbf{Z}_{U\ 13})$ is curve F.

of four parallel 79.7 mm long dipole antennas (side-by-side configuration). The radius of the array is 47.8 mm, so that it presents a 67.7 mm spacing between the nearest array elements. Each antenna has a 54 mm long feeder. At the center frequency $f_c = 1880$ MHz, $\mathbf{Z}_{Sant}$ is approximately given by:

$$\mathbf{Z}_{Sant} \approx \begin{pmatrix} 134+44j & -40-45j & -16+27j & -40-45j \\ -40-45j & 134+44j & -40-45j & -16+27j \\ -16+27j & -40-45j & 134+44j & -40-45j \\ -40-45j & -16+27j & -40-45j & 134+44j \end{pmatrix} \Omega \tag{26}$$

Three entries of $\mathbf{Z}_{Sant}$ are plotted as a function of frequency in Fig. 8. At any frequency, $\mathbf{Z}_{Sant}$ is symmetric and circulant, as shown in (26), so that all entries of $\mathbf{Z}_{Sant}$ are plotted in Fig. 8. All numerical results presented in this section were obtained with a numerical analysis program, using the induced emf method to compute the impedance matrix of the antenna array [37, ch. 14].

We want to compute the component values to obtain $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$. After some iterations (manual trial and errors), the value selected at step 1 is

$$\mathbf{C}_A = \begin{pmatrix} 10.20 & -2.10 & -1.20 & -2.10 \\ -2.10 & 10.20 & -2.10 & -1.20 \\ -1.20 & -2.10 & 10.20 & -2.10 \\ -2.10 & -1.20 & -2.10 & 10.20 \end{pmatrix} \text{pF} \tag{27}$$

for which we obtain

$$\mathbf{C}_U \approx \begin{pmatrix} 19.54 & -6.94 & -0.19 & -6.94 \\ -6.94 & 19.54 & -6.94 & -0.19 \\ -0.19 & -6.94 & 19.54 & -6.94 \\ -6.94 & -0.19 & -6.94 & 19.54 \end{pmatrix} \text{pF} \tag{28}$$

at step 2 and

$$\mathbf{L} \approx \begin{pmatrix} 1.276 & 0.397 & 0.285 & 0.397 \\ 0.397 & 1.276 & 0.397 & 0.285 \\ 0.285 & 0.397 & 1.276 & 0.397 \\ 0.397 & 0.285 & 0.397 & 1.276 \end{pmatrix} \text{nH} \tag{29}$$

at step 3. The value of $\mathbf{L}$ given by (29) can be used to design the windings shown in Fig. 7. The values of $\mathbf{C}_A$ and $\mathbf{C}_U$ given by (27) and (28) can be used to design the network of adjustable impedance devices.

For $\mathbf{C}_A$, $\mathbf{L}$ and $\mathbf{C}_U$ given by (27)–(29), three entries of $\mathbf{Z}_U$ are plotted as a function of frequency in Fig. 9. At any frequency, $\mathbf{Z}_U$ being symmetric and circulant, all entries of $\mathbf{Z}_U$ are in fact plotted in Fig. 9. The plot clearly shows that $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ is obtained at $f_c = 1880$ MHz.

Instead of plotting the real and imaginary parts of the entries of $\mathbf{Z}_{Sant}$ and $\mathbf{Z}_U$, it is possible to use a scalar figure of merit such as the return figure $F(\mathbf{Z})$ given by

$$F(\mathbf{Z}) = |||\mathbf{S}(\mathbf{Z})|||_2 \tag{30}$$

where $\mathbf{Z}$ is an impedance matrix of size $q \times q$, $\mathbf{S}(\mathbf{Z})$ is a scattering matrix defined by

$$\mathbf{S}(\mathbf{Z}) = (\mathbf{Z} + r_0\mathbf{I}_q)^{-1}(\mathbf{Z} - r_0\mathbf{I}_q) = (\mathbf{Z} - r_0\mathbf{I}_q)(\mathbf{Z} + r_0\mathbf{I}_q)^{-1} \tag{31}$$

and the spectral norm $|||\mathbf{A}|||_2$ of a square matrix $\mathbf{A}$ is the square root of the largest eigenvalue of $\mathbf{A}\mathbf{A}^*$, and the largest singular value of $\mathbf{A}$ [36, § 5.6.6 and § 7.3.10]. Some properties of $F(\mathbf{Z})$ are presented in the Appendix. $F(\mathbf{Z})$ expressed in decibels is $F_{dB} = 20\ \log(F(\mathbf{Z}))$, where log is the decimal logarithm. By (35), it follows that $F_{dB}(\mathbf{Z}_U) \leq 0$ dB and $F_{dB}(\mathbf{Z}_{Sant}) \leq 0$ dB. An ideal match corresponds to $F_{dB}(\mathbf{Z}_U) = -\infty$ dB.

$F_{dB}(\mathbf{Z}_U)$ and $F_{dB}(\mathbf{Z}_{Sant})$ are plotted as a function of frequency in Fig. 10. It confirms that $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ is obtained at $f_c = 1880$ MHz. Fig. 9 and Fig. 10 have been obtained for the initial values of $\mathbf{L}$ and $\mathbf{C}_U$ given by (28), (29), which were computed to obtain $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ at $f_c = 1880$ MHz, for $\mathbf{C}_A$ given by (27). This does not prove that the antenna tuner can provide the desired tunability, when, as shown in Fig. 7, $\mathbf{L}$ is fixed while $\mathbf{C}_U$ and $\mathbf{C}_A$ may be varied. Unlike what was done in Section II, we cannot plot $D_{UR}(\mathbf{Z}_{Sant})$ and $D_{UTR}(\mathbf{Z}_U)$ corresponding to the $p = 20$ sets of the achievable values of the real variables $X_1, \ldots, X_p$, for two reasons: $D_{UR}(\mathbf{Z}_{Sant})$ and $D_{UTR}(\mathbf{Z}_U)$ are subsets of $S(4)$, a real vector space of dimension 20; and, if each adjustable impedance device shown in Fig. 7 provides 100 capacitance values, there are 10^{40} different circuits to consider. However, numerical partial derivatives applied to (20) and a matrix rank computation can be used to show that, at the point defined by (27)–(29), the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is $m(m+1) = 20$. By Proposition 6, it follows that the LD of

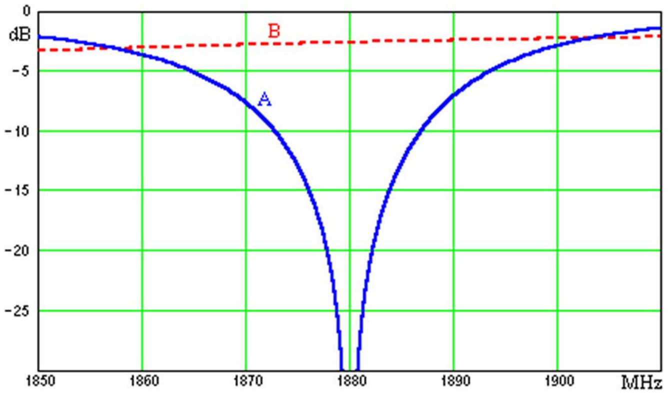

Fig. 10. The return figure versus frequency: F_{dB} $(\mathbf{Z}_U)$ for $\mathbf{Z}_U$ shown in Fig. 9 is curve A and F_{dB} $(\mathbf{Z}_{Sant})$ is curve B.

$D_{UTR}(\mathbf{Z}_U)$ is also $n(n+1) = 20$. By Proposition 8, it follows that the antenna tuner has a full tuning capability.

To confirm the effectiveness of the antenna tuner, we consider a variation in $\mathbf{Z}_{Sant}$ and $\mathbf{Z}_U$ caused by a variation in frequency. $\mathbf{L}$ being fixed, the antenna tuner performs as intended if realizable new values of $\mathbf{C}_U$ and $\mathbf{C}_A$ exist, which provide $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ at the new frequency. A possible new value of $\mathbf{C}_A$ is given by

$$\omega \mathbf{C}_A = (\omega \mathbf{L})^{-1} - \mathbf{B}_A + \mathbf{G}_A \left[(g_0 \mathbf{G}_A)^{-1} (\omega \mathbf{L})^{-2} - \mathbf{1}_m \right]^{\frac{1}{2}} \tag{32}$$

and (22) can be used to obtain the new value of $\mathbf{C}_U$. Thus, $\mathbf{L}$ being given by (29), we have determined new values of $\mathbf{C}_U$ and $\mathbf{C}_A$ such that $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ at 1860 MHz:

$$\mathbf{C}_A \approx \begin{pmatrix} 10.59 & -2.36 & -1.38 & -2.36 \\ -2.36 & 10.59 & -2.36 & -1.38 \\ -1.38 & -2.36 & 10.59 & -2.36 \\ -2.36 & -1.38 & -2.36 & 10.59 \end{pmatrix} \text{pF} \tag{33}$$

and

$$\mathbf{C}_U \approx \begin{pmatrix} 19.13 & -6.48 & -0.18 & -6.48 \\ -6.48 & 19.13 & -6.48 & -0.18 \\ -0.18 & -6.48 & 19.13 & -6.48 \\ -6.48 & -0.18 & -6.48 & 19.13 \end{pmatrix} \text{pF} \tag{34}$$

For these new values of $\mathbf{C}_U$ and $\mathbf{C}_A$, three entries of $\mathbf{Z}_U$ are plotted as a function of frequency in Fig. 11 and $F_{dB}(\mathbf{Z}_U)$ is plotted as a function of frequency in Fig. 12. The wanted result $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ at 1860 MHz is indeed achieved. This is a consequence of the full tuning capability.

Lastly, we note that, in Fig. 10 and Fig. 12, the bandwidth for $F_{dB}(\mathbf{Z}_U) < -15$ dB is significantly narrower than the frequency band of intended operation (1850 MHz to 1910 MHz). This is not necessarily a drawback because, unlike a matching network, an antenna tuner is meant to be tuned.

VII. Application to the Analysis of Antenna Tuners

To analyze a reciprocal MAPMUP antenna tuner structure, a mapping f_U defined by (3) can always be determined. Based on Definition 1 of Section III, the LD of $D_{UR}(\mathbf{Z}_{Sant})$ can then be easily computed as the rank of $\partial f_U / \partial X_1, \ldots, \partial f_U / \partial X_p$, using numerical partial derivatives. This is for instance what was done in Section VI, where f_U was obtained from (20). It is then possible to use Proposition 8 to determine whether the antenna tuner has a full tuning capability. This characteristic does not take into account the availability of a sufficient number of

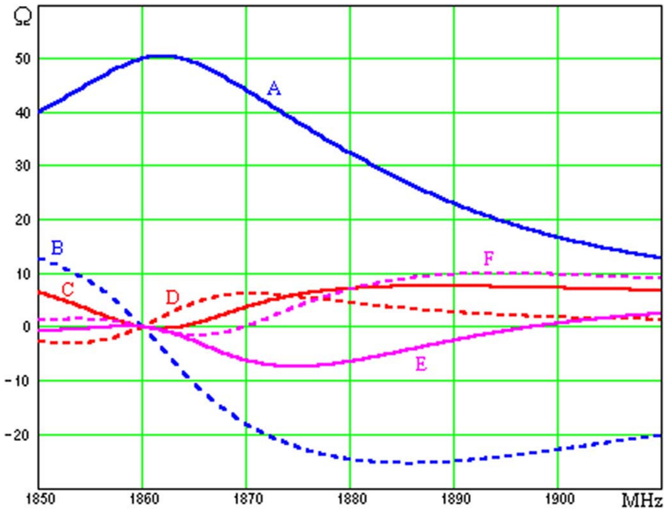

Fig. 11. $\mathbf{Z}_U$ for the new values of $\mathbf{C}_A$ and $\mathbf{C}_U$: $\text{Re}(\mathbf{Z}_{U\ 11})$ is curve A; $\text{Im}(\mathbf{Z}_{U\ 11})$ is curve B; $\text{Re}(\mathbf{Z}_{U\ 12})$ is curve C; $\text{Im}(\mathbf{Z}_{U\ 12})$ is curve D; $\text{Re}(\mathbf{Z}_{U\ 13})$ is curve E; and $\text{Im}(\mathbf{Z}_{U\ 13})$ is curve F.

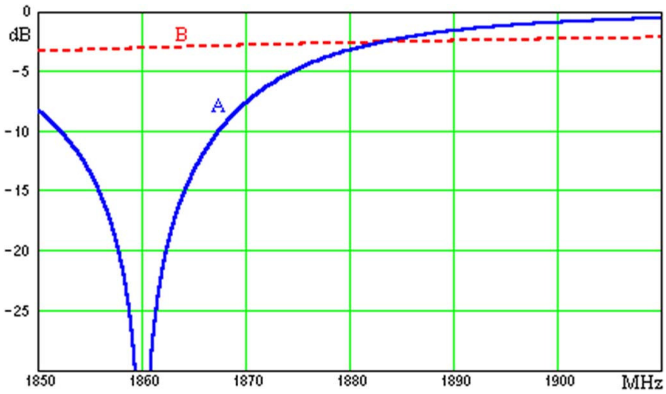

Fig. 12. Return figure versus frequency: F_{dB} $(\mathbf{Z}_U)$ for $\mathbf{Z}_U$ shown in Fig. 11 is curve A and F_{dB} $(\mathbf{Z}_{Sant})$ is curve B.

elements of $D_{UR}(\mathbf{Z}_{Sant})$ and of $D_{UTR}(\mathbf{Z}_U)$, where these elements are most needed, but it is nevertheless a useful indication of what the antenna tuner could do, or cannot do.

If $p < m(m+1)$, no computation is necessary to determine that an antenna tuner does not have a full tuning capability. In Section V, we also faced a similar situation when we studied the uncoupled SAPSUP antenna tuners shown in Fig. 6.

In the example of Section VI, we have used the closed-form expressions (22) and (32) to compute the reactances $X_1, \ldots, X_p$ of the adjustable impedance devices, which were used to obtain Figs. 11–12 and validate the capability of the MAPMUP antenna tuner. It must be stressed that such closed-form expressions are usually not available, for instance when losses are taken into account, or when the circuit of the antenna tuner is too complex. In this case, a numerical computation of $X_1, \ldots, X_p$ is possible but not very easy because the problem is not linear. This is why the computation of the LD of $D_{UR}(\mathbf{Z}_{Sant})$ and Proposition 8 are very useful.

VIII. Conclusion

We have defined, for the most general reciprocal MAPMUP antenna tuner, a user port impedance range $D_{UR}(\mathbf{Z}_{Sant})$ and a user port tuning range $D_{UTR}(\mathbf{Z}_{UW})$. We have also defined their LDs and proved some relations between them. These new figures of merit are easily computed and indicative of the antenna tuner's ability to compensate small deviations in $\mathbf{Z}_U$ or $\mathbf{Z}_{Sant}$.

Our main result is that the LD of $D_{UR}(\mathbf{Z}_{Sant})$ is equal to the maximum possible value $m\ (m+1)$ if and only if the antenna tuner has a full tuning capability. This property can be easily applied to any reciprocal MAPMUP antenna tuner. It was used to show that, for $n \geq 3$, a reciprocal MAPMUP antenna tuner made of n uncoupled SAPSUP antenna tuners does not have a full tuning capability.

We have also presented a new MAPMUP antenna tuner which cannot be separated into uncoupled SAPSUP antenna tuners. It has the structure of a multidimensional π-network. For n antenna ports and n user ports, the value of the LD of $D_{UR}(\mathbf{Z}_{Sant})$ indicates that this antenna tuner has a full tuning capability. This is confirmed by a computation showing that the antenna tuner can provide a wanted diagonal impedance matrix at different frequencies.

APPENDIX

Let us review some properties of the return figure $F(\mathbf{Z})$ defined by (30) and (31). If $\mathbf{Z}$ is the impedance matrix of a passive device, it is well known that $\mathbf{1}_q - \mathbf{S}(\mathbf{Z})\mathbf{S}(\mathbf{Z})^*$ is positive semidefinite, so that by [36, Corollary 7.7.4], we conclude that

$$0 \leq F(\mathbf{Z}) \leq 1. \tag{35}$$

By (30), an ideal match corresponds to $F(\mathbf{Z}) = 0$. Let $\mathbf{a}$ be a normalized incident voltage wave measured in q uncoupled 2-conductor transmission lines of characteristic impedance r_0. We know that $\mathbf{b} = \mathbf{S}\mathbf{a}$ is the normalized reflected wave produced by a load presenting the impedance matrix $\mathbf{Z}$, and that, since r_0 is real, the Euclidian vector norms $||\mathbf{a}||_2$ and $||\mathbf{b}||_2$ are the incident and reflected powers, respectively. The spectral norm being the matrix norm induced by the Euclidian vector norm, it follows that

$$||\mathbf{b}||_2 \leq F(\mathbf{Z})||\mathbf{a}||_2 \tag{36}$$

and

$$F(\mathbf{Z}) = \max_{\mathbf{a}\neq 0} \frac{||\mathbf{b}||_2}{||\mathbf{a}||_2} \tag{37}$$

We note that $F(\mathbf{Z})$ is different from the normalized total multiport reflectance defined and used in [38] and [39], the definition of which uses the Frobenius norm of $\mathbf{S}(\mathbf{Z})$. The Frobenius norm of $\mathbf{S}(\mathbf{Z})$ is greater than or equal to $F(\mathbf{Z})$ and it need not be less than or equal to 1 for a passive device. Moreover, the Frobenius norm is not an induced norm.

REFERENCES

[1] M. Thompson and J. K. Fidler, "Determination of the impedance matching domain of impedance matching networks," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 51, no. 10, pp. 2098–2106, Oct. 2004.

[2] E. L. Firrao, A.-J. Annema, and B. Nauta, "An automatic antenna tuning system using only RF signal amplitudes," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 55, no. 9, pp. 833–837, Sep. 2008.

[3] C. Hoarau, N. Corrao, J.-D. Arnould, P. Ferrari, and P. Xavier, "Complete design and measurement methodology for a tunable RF impedance-matching network," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 11, pp. 2620–2627, Nov. 2008.

[4] L. Huang and P. Russer, "Electrically tunable antenna design: Procedure for mobile applications," *IEEE Trans. Microw. Theory Tech.*, vol. 56, no. 12, pp. 2789–2797, Dec. 2008.

[5] P. Ramachandran, Z. D. Milosavljevic, and C. Beckman, "Adaptive matching circuitry for compensation of finger on handset antennas," in *Proc. 3rd Eur. Conf. Antenna Propag. (EuCAP)*, Mar. 2009, pp. 801–804.

[6] M. A. de Jongh, A. van Bezooijen, K. R. Boyle, and T. Bakker, "Mobile phone performance improvements using an adaptively controlled antenna tuner," *2011 IEEE MTT-S Int. Microw. Symp. Dig.*, pp. 1–4, Jun. 2011.

[7] F. Chan Wai Po, E. de Foucauld, D. Morche, P. Vincent, and E. Kerhervé, "A novel method for synthesizing an automatic matching network and its control unit," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 58, no. 9, pp. 2225–2236, Sep. 2011.

[8] F. Sonnerat, R. Pilard, F. Gianesello, F. Le Pennec, Ch. Person, and D. Gloriat, "4 G antenna tuner integrated in a 130 nm CMOS SOI technology," in *Proc. 2012 IEEE 12th Topical Meet. Silicon Monolithic Integr. Circuits RF Syst. (SiRF)*, Jan. 2012, pp. 191–194.

[9] R. Valkonen, C. Icheln, and P. Vainikainen, "Power dissipation in mobile antenna tuning circuits under varying impedance conditions," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 37–40, 2012.

[10] N. J. Smith, C.-C. Chen, and J. L. Volakis, "An improved topology for adaptive agile impedance tuners," *IEEE Antennas Wireless Propag. Lett.*, vol. 12, pp. 92–95, Mar. 2013.

[11] S. Stein, "On cross coupling in multiple-beam antennas," *IRE Trans. Antennas Propag.*, vol. 10, no. 5, pp. 548–557, Sep. 1962.

[12] C. A. Desoer, "The maximum power transfer theorem for n-ports," *IEEE Trans. Circuit Theory*, vol. CT-20, no. 3, pp. 328–330, May 1973.

[13] J. W. Wallace and M. A. Jensen, "Termination-dependent diversity performance of coupled antennas: Network theory analysis," *IEEE Trans. Antennas Propag.*, vol. 52, no. 1, pp. 98–105, Jan. 2004.

[14] M. L. Morris and M. A. Jensen, "Network model for MIMO systems with coupled antennas and noisy amplifiers," *IEEE Trans. Antennas Propag.*, vol. 53, no. 1, pp. 545–552, Jan. 2005.

[15] F. Broydé and E. Clavelier, "Taking advantage of mutual coupling in radio communication systems using a multiport antenna array," *IEEE Antennas Propag. Mag.*, vol. 49, no. 4, pp. 208–220, Aug. 2007.

[16] R. Mohammadkhani and J. S. Thompson, "Adaptive uncoupled termination for coupled arrays in MIMO systems," *IEEE Trans. Antennas Propag.*, vol. 61, no. 8, pp. 4284–4295, Aug. 2013.

[17] M. A. Jensen and B. Booth, "Optimal uncoupled impedance matching for coupled MIMO arrays," in *Proc. 1st Eur. Conf. Antennas Propag. (EuCAP)*, Nov. 2006, pp. 1–4.

[18] Y. Fei, Y. Fan, B. K. Lau, and J. S. Thompson, "Optimal singleport matching impedance for capacity maximization in compact MIMO arrays," *IEEE Trans. Antennas Propag.*, vol. 56, no. 11, pp. 3566–3575, Nov. 2008.

[19] B. K. Lau and J. B. Andersen, "On closely coupled dipoles with load matching in a random field," in *Proc. IEEE 17th Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC)*, Sep. 2006, pp. 1–5.

[20] B. K. Lau, J. B. Andersen, A. F. Molisch, and G. Kritensson, "Antenna matching for capacity maximization in compact MIMO systems," in *Proc. 3rd Int. Symp. Wireless Commun. Syst. (ISWCS)*, Sep. 2006, pp. 253–257.

[21] J. B. Andersen and B. K. Lau, "On closely coupled dipoles in a random field," *IEEE Antennas Wireless Propag. Lett.*, vol. 5, pp. 73–75, 2006.

[22] M. A. Jensen and B. K. Lau, "Uncoupled matching for active and passive impedances of coupled arrays in MIMO systems," *IEEE Trans. Antennas Propag.*, vol. 58, no. 10, pp. 3336–3343, Oct. 2010.

[23] N. Murtaza, M. Hein, and E. Zameshaeva, "Reconfigurable decoupling and matching network for a cognitive Antenna," in *Proc. 41st Eur. Microw. Conf. (EuMC)*, Oct. 2011, pp. 874–877.

[24] X. Tang, K. Mouthaan, and J. C. Coetzee, "Tunable decoupling and matching network for diversity enhancement of closely spaced antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 268–271, 2012.

[25] A. Krewski and W. L. Schroeder, "Electrically tunable mode decomposition network for 2-port MIMO antennas," in *Proc. Loughborough Antennas Propag. Conf. (LAPC)*, Nov. 2013, pp. 553–558.

[26] STHVDAC-303 High voltage BST capacitance controller, Datasheet, STMicroelectronics, Doc. ID 18317 Rev 3, Nov. 2012.

[27] STPTIC Parascan tunable integrated capacitor, Datasheet, STMicroelectronics, Doc. ID 023772 Rev 3, Jan. 2014.

[28] PE64904 UltraCMOS Digitally Tunable Capacitor (DTC) 100–3000 MHz, Datasheet, Peregrine Semiconductor, Doc. No. 70-0325-06, 2012.

[29] RF1109 5 bit PAC (Programmable Array of Capacitors), Datasheet, RF Micro Devices Inc., DS140106, 2013.

[30] R. E. Collin, *Antennas and Radiowave propagation*, Int. ed. New York: McGraw-Hill, 1985.

[31] F. Broydé and E. Clavelier, "The noise performance of a multiple-input-port and multiple-output-port low-noise amplifier connected to an array of coupled antennas," *Int. J. Antennas Propag.*, vol. 2011, Nov. 2011, Art. ID 438478.

[32] J. Weber, C. Volmer, K. Blau, R. Stephan, and M. A. Hein, "Miniaturisation of antenna arrays for mobile communications," in *Proc. 35th Eur. Microw. Conf.*, Paris, France, Oct. 2005, pp. 1173–1176.
[33] J. Weber, C. Volmer, K. Blau, R. Stephan, and M. A. Hein, "Miniaturized antenna arrays using decoupling networks with realistic elements," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 6, pp. 2733–2740, Jun. 2006.
[34] J. Weber, C. Volmer, K. Blau, R. Stephan, and M. A. Hein, "Implementation of a miniaturized antenna array with predefined orthogonal radiation patterns," in *Proc. First Eur. Conf. Antennas Propag. (EuCAP)*, Nice, France, Nov. 2006, pp. 1–5.
[35] Y. Sun and J. K. Fidler, "Design of Π impedance matching networks," in *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, Jun. 1994, vol. 5, pp. 5–8.
[36] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.
[37] E. C. Jordan and K. G. Balmain, *Electromagnetic Waves and Radiating Systems*, 2nd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 1968.
[38] W. L. Schroeder and A. Krewski, "Total multi-port return loss as a figure of merit for MIMO antenna systems," in *Proc. 3rd Eur. Wireless Technol. Conf. (EuWiT)*, Sep. 2010, pp. 265–268.
[39] A. Krewski, W. L. Schroeder, and K. Solbach, "Matching network synthesis for mobile MIMO antennas based on minimization of the total multi-port reflectance," in *Proc. Loughborough Antenna Propag. Conf. (LAPC)*, Nov. 2011, pp. 1–4.

Frédéric Broydé (S'84–M'86–SM'01) was born in France in 1960. He received the M.S. degree in physics engineering from the Ecole Nationale Supérieure d'Ingénieurs Electriciens de Grenoble, France (ENSIEG) and the Ph.D. in microwaves and microtechnologies from the Université des Sciences et Technologies de Lille, France (USTL).

He co-founded Excem, Maule, France, in May 1988, a company providing engineering and research and development services. He is president and CTO of Excem. His most active research areas are multiconductor transmission line theory, the development of advanced schemes for high-speed interconnects and wireless transmission systems. He served as program chair of SPI 2013, the conference on signal and power integrity organized by the IEEE in Europe. Dr. Broydé is a radio amateur (F5OYE). He is author or co-author of about 90 technical papers, and inventor or co-inventor in patent applications for about 60 inventions for which about 25 U.S. patents have been granted. A complete bibliography including published papers and patents applications of Dr. Broydé is available on the http://www.eurexcem.com web site.

Evelyne Clavelier (S'84–M'85–SM'02) was born in France in 1961. She received the M.S. degree in physics engineering from the Ecole Nationale Supérieure d'Ingénieurs Electriciens de Grenoble, France (ENSIEG). She is co-founder of Excem, Maule, France. She is CEO of Excem. She is also Manager of Eurexcem (a subsidiary of Excem) and President of Tekcem, a company selling or licensing intellectual property rights. She has been active in the field of signal integrity, in which she studied modal signaling and pseudo-differential signaling. Her current research area is antenna tuning for MIMO wireless handsets. Ms. Clavelier is a radio amateur (F1PHQ). She is author or co-author of about 75 technical papers. A complete bibliography of Ms. Clavelier is available on the http://www.eurexcem.com web site.