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MINIMUM ATTENUATION AND INPUT IMPEDANCE DOMAIN OF A LINEAR FILTER

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It is a well established fact that the behaviour and performance of linear filters strongly depend on the impedance connected at its input and output. This observation is of utmost importance when dealing with power line filters.

Classically, the characterization of a linear filter is done with the use of a quantity known as insertion loss. We first introduce this quantity together with three other important concepts, namely the total attenuation, the absorption attenuation and the mismatch attenuation.

We then define what we call the worst case at the output and the worst case at the input of the filter. We show how to compute insertion loss and attenuations in a worst case.

The minimum attenuation of the filter is the absorption attenuation in the worst case (at the output) and is also the total attenuation in the worst case at the output and at the input.

The input impedance domain of a filter is defined and shown to be a convenient tool to understand the filter's behaviour and to compute the mismatch attenuation in a worst case.

We show that in many cases, the most relevant quantity for the study of a power line filter is the total attenuation, which can be most easily assessed with the combined use of minimum attenuation and input impedance domain.

We compare these new developments in power line filter theory to former studies, which involved the worst case insertion loss or minimum voltage attenuation.

INTRODUCTION

This paper deals with the theory of specification, design and characterization of linear filters connected to loads or sources of unknown impedances. This topic is especially relevant to the study of power line filters used to obtain electromagnetic compatibility from equipment connected to the power network.

We know that circuit theory allows one to analyze and synthesize complex linear filters, and thus makes it possible to meet almost any needs of electrical or electronics engineers. However, the elegant and subtle theory involved in those design techniques, known as classic filtering theory, is based on the knowledge of the load and source impedances to be connected to the filter under consideration [1]. Moreover, methods used for synthesis also require those impedances to be purely resistive.

Unfortunately, power line filters are meant to be inserted between an electrical appliance and a power network: power networks exhibit impedances which, above some 10 kHz, are usually time varying, frequency dependant and reactive [4], [5], [6], and the behaviour of appliances may have a similar complexity. As power and voltages at the input and output of the filter strongly depend on those impedances, it becomes apparent that a filter cannot be characterized with a single constant and real parameter at each frequency.

The use of the insertion loss of a filter, as measured between a 50 Ohm source and a 50 Ohm load, for the characterization of a power line filter in their stop band is therefore meaningless from an EMC point of view. This point is now recognized in most major standards on filters [15], [16], [17], but has been disregarded by many authors, EMC engineers, and filter manufacturers: even though alternative characterization means have been proposed, their use remain unusual.

WORST-CASE METHODS FOR FILTER CHARACTERIZATION

Two concepts have been proposed for the characterization of power line filters, namely : the minimum voltage attenuation [2], [9], [16], the worst-case insertion loss [3], [4], [7], [10]. We will carefully define those quantities in mathematical terms along the paper, but for our present purpose, it is sufficient to say that they both resort to the notion of worst case behaviour:

- in the stop band of the filter, the greater voltage attenuation and insertion loss, the better the filter,

- the "worst-case" or "minimum" value of those quantities correspond to their smallest possible value when the load or source impedance take on all possible values in the positive real part half complex plane.

In the following, three other new worst-case quantities will be introduced, together with the two classical ones.

ASSUMPTIONS AND NOTATIONS

Let us consider a linear, passive and reciprocal two-port. We will use the notations of figure 1, in which the source connected to the input of the filter is sinusoidal, of radian frequency ω, linear, of open circuit voltage v_S, and has an impedance Z_S. The load will also be assumed linear and has an impedance Z_L.

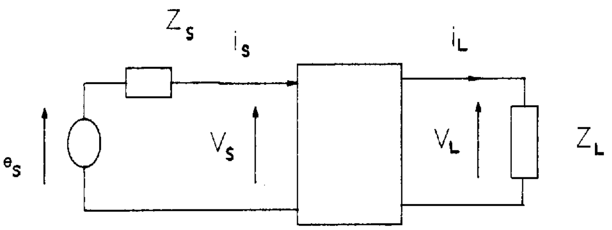

Figure 1 : Notations

The filter under study is, at any radian frequency ω, completely characterized by its chain matrix A, which is defined by:

$$\begin{pmatrix} v_L \\ i_L \end{pmatrix} = A \begin{pmatrix} v_S \\ i_S \end{pmatrix} \qquad \ldots(1)$$

and

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad \ldots(2)$$

where v_S, i_S, v_L and i_L are complex rms values. The coefficients of the chain matrix are also complex numbers, and we write

$$\forall(\alpha,\beta) \in \{1,2\}^2 \qquad a_{\alpha\beta} = b_{\alpha\beta} + j\,c_{\alpha\beta} \qquad \ldots(3)$$
$$(b_{\alpha\beta}, c_{\alpha\beta}) \in \mathbb{R}^2$$

We may then define several powers:

the input power, $P_S = Re\,(v_S \bar{i}_S)$ ...(4)

the output power, $P_L = Re\,(v_L \bar{i}_L)$ ...(5)

the available power, P_{MAX}, which is the maximum power which the source may produce

$$P_{MAX} = \frac{|e_S|^2}{4\,Re(Z_S)} \qquad \ldots(6)$$

the power without filter, P_{WF}, which is the power received by the load when it is directly connected to the source

$$P_{WF} = |e_S|^2 \frac{Re\,(Z_L)}{|Z_L + Z_S|^2} \qquad \ldots(7)$$

INSERTION LOSS AND VOLTAGE ATTENUATION

Insertion loss I is defined by

$$I = \sqrt{\frac{P_{WF}}{P_L}} \qquad \ldots(8)$$

this expression being equivalent to

$$I = \left| \frac{v_{WF}}{v_L} \right| \qquad \ldots(9)$$

where v_{WF} is the voltage across the load when it is directly connected to the source. A straightforward calculation yields

$$I = \left| \frac{a_{11} Z_S + a_{22} Z_L + a_{21} Z_L Z_S + a_{12}}{Z_L + Z_S} \right| \qquad \ldots(10)$$

This formula shows explicitly that I is a function of Z_S and Z_L. For this reason, we will sometimes write

$$I = I(Z_S, Z_L) \qquad \ldots(11)$$

Insertion loss versus frequency data, usually presented on a plot, are the most widely used way of characterizing a filter.

Historically, Mil Std 220 A [15] proposed the evaluation of power line filter through an insertion loss measurement in a 50 ohms system. The same standard was later (1978) modified in response to the outrage of many prominent EMC experts: it was admitted that the 50 Ω -insertion loss did not correctly characterize the filter. Nevertheless this technique was kept as a quality control tool, and no replacement was offered.

Voltage attenuation α_V is defined as

$$\alpha_V = \left| \frac{v_S}{v_L} \right| \qquad \ldots(12)$$

this expression is easily transformed into

$$\alpha_V = \left| a_{22} - \frac{a_{12}}{Z_L} \right| \qquad \ldots(13)$$

which shows that α_V is a function of Z_L but not of Z_S.

POWER ATTENUATIONS

We will now introduce three (more or less) new quantities.

The total attenuation $\mathcal{A}$ is defined as

$$\mathcal{A} = \sqrt{\frac{P_{MAX}}{P_L}} \qquad \ldots(14)$$

it can be computed from the A matrix with

$$\mathcal{A} = \frac{|a_{11} Z_S + a_{22} Z_L + a_{21} Z_L Z_S + a_{12}|}{2\sqrt{Re(Z_S)\;Re\,(Z_L)}} \qquad \ldots(15)$$

This formula shows that $\mathcal{A}$ is a function of Z_S and Z_L, which we will occasionally write

$$\mathcal{A} = \mathcal{A}\,(Z_S, Z_L) \qquad \ldots(16)$$

The absorption attenuation $\mathfrak{A}$ is defined as

$$\mathfrak{A} = \sqrt{\frac{P_S}{P_L}} \qquad \ldots(17)$$

and can be computed from the A matrix with

$$\mathfrak{A} = \left(\frac{Re\left([a_{11} - Z_L a_{21}]\,\overline{[a_{22} Z_L - a_{12}]}\right)}{Re\,(Z_L)} \right)^{1/2} \qquad \ldots(18)$$

where it becomes apparent that $\mathfrak{A}$ is a function of Z_L but not of Z_S. We will therefore sometimes write

$$\mathfrak{A} = \mathfrak{A}(Z_L) \qquad \ldots(19)$$

The mismatch attenuation $\mathfrak{B}$ is defined as

$$\mathfrak{B} = \sqrt{\frac{P_{MAX}}{P_S}} \qquad \ldots(20)$$

and can be computed with ...(21)

$$\mathfrak{B} = \frac{|a_{11} Z_S + a_{22} Z_L - a_{21} Z_L Z_S - a_{12}|}{2\sqrt{Re(Z_S)}\sqrt{Re([a_{11}-Z_L a_{21}][\bar{a}_{22}\bar{Z}_L - \bar{a}_{12}])}}$$

This quantity appears to be dependant on both Z_S and Z_L, which will allow us to write

$$\mathfrak{B} = \mathfrak{B}(Z_S, Z_L) \quad \text{...(22)}$$

when necessary.

The three power attenuations presented above are all greater than 1, and are related by:

$$A = \mathfrak{A} \cdot \mathfrak{B} \quad \text{...(23)}$$

None of those three attenuations is the attenuation a commonly used in the theory of linear network [1], [8], and defined as

$$e^{-a} = |S_{21}| \quad \text{...(24)}$$

where S_{21} is the transmittance between the source and the load, relatively to the reference resistances R_1 and R_2.

The total attenuation A is more general, and satisfies

$$A = e^{a} \qquad A_{dB} \simeq 0{,}434\, a \quad \text{...(25)}$$

when source and load impedances are purely resistive and respectively equal to $Z_S = R_1$ and $Z_L = R_2$.

For $Z_S = \bar{Z}_L$, it can be seen that

$$A = I \quad \text{...(26)}$$

and there is therefore no need to distinguish between insertion loss and total attenuation when source and load are purely resistive.

INSERTION LOSS AND ATTENUATIONS IN A WORST CASE

For all five quantities introduced (insertion loss, voltage attenuation and power attenuations), and at frequencies inside the stop band of the filter, the greater a given quantity, the better the filter. If load or source impedance are made random, the worst case will therefore happen when a given quantity reaches a minimum value.

As impedances only take on values with a positive real part, we find it convenient to note

$$\mathbb{C}_+ = \mathbb{R}_+ + j\,\mathbb{R} \quad \text{...(27)}$$

the positive real part half complex plane.

For a quantity X dependant on Z_L, but not on Z_S (in the present instance X is either α_v or $\mathfrak{A}$), we define X in the worst case as

$$X_{min} = Inf\{X(Z_L) \mid Z_L \in \mathbb{C}_+\} \quad \text{...(28)}$$

and we call it minimum X.

Thus, $\alpha_{v\,min}$ is the minimum voltage attenuation. The use of this quantity is advised in a C.I.S.P.R. standard [16].

In a similar way, $\mathfrak{A}_{MIN}$ is the minimum absorption attenuation. As this quantity exhibits very particular properties, we will simply call it minimum attenuation and use the symbol M instead of $\mathfrak{A}_{MIN}$:

$$M = Inf\{\mathfrak{A}(Z_L) \mid Z_L \in \mathbb{C}_+\} \quad \text{...(29)}$$

For a quantity X dependant on both Z_S and Z_L (in the present instance X is either A or $\mathfrak{B}$ or I), we define X in the worst case at the input as

$$X_i(Z_L) = Inf\{X(Z_S, Z_L) \mid Z_S \in \mathbb{C}_+\} \quad \text{...(30)}$$

and X in the worst case at the output as

$$X_o(Z_S) = Inf\{X(Z_S, Z_L) \mid Z_L \in \mathbb{C}_+\} \quad \text{...(31)}$$

We notice that X_i is a function of Z_L and that X_o is a function of Z_S. It is therefore possible to consider X in the worst case at the input and at the output, which is a constant and is denoted by X_{min}.

CALCULATION OF WORST CASES

Calculation of insertion loss in the worst case at the input and of insertion loss in the worst case at the output was already addressed by Audone and Bolla [10].

Calculation of minimum voltage attenuation was already carried out by Hinton et al, and by Jarvis and Thomson [2], [9]. Their calculation is contained in C.I.S.P.R. 17 [16]. A computer program was also dedicated to this calculation [11], [12], this work being done by Jones et al.

Calculation of total attenuation in the worst case at the output turns out to be relatively simple, and one finds

$$A_o(Z_S) = \left(\frac{Re([a_{22} - Z_S a_{21}][\bar{a}_{11}\bar{Z}_S - \bar{a}_{12}])}{Re(Z_S)}\right)^{1/2} \quad \text{...(32)}$$

From the symetry of equation (15) it is obvious that total attenuation in the worst case at the input is obtained by interchanging a_{22} and Z_S with a_{11} and Z_L in (32):

$$A_i(Z_L) = \left(\frac{Re([a_{11} - Z_L a_{21}][\bar{a}_{22}\bar{Z}_L - \bar{a}_{12}])}{Re(Z_L)}\right)^{1/2} \quad \text{...(33)}$$

Comparing (18) and (33) proves that

$$\mathfrak{A}(Z_S) = A_i(Z_S) \quad \text{...(34)}$$

and also

$$M = A_{min} \quad \text{...(35)}$$

Calculation of minimum attenuation is slightly more complex. We first introduce four new parameters:

$$k = b_{21}b_{12} + c_{21}c_{12} + b_{11}b_{22} + c_{11}c_{22} \quad \text{...(36)}$$

$$\ell = c_{11}b_{22} + b_{21}c_{12} - b_{11}c_{22} - c_{21}b_{12} \quad \text{...(37)}$$

$$m = -(b_{21}b_{22} + c_{21}c_{22}) \quad \text{...(38)}$$

$$n = -(b_{11}b_{12} + c_{11}c_{12}) \quad \text{...(39)}$$

One can prove that those coefficients satisfy the following relations:

$$k^2 - 4mn + \ell^2 = 1 \quad \text{...(40)}$$

$$m \geq 0 \quad \text{...(41)}$$

$$n \geq 0 \quad \text{...(42)}$$

$$4mn - \ell^2 \geq 0 \quad \text{...(43)}$$

Minimum attenuation is then given by:

$$M = \sqrt{k + \sqrt{4mn - \ell^2}} \quad \text{...(44)}$$

Calculation of mismatch attenuation in the worst case at the output could be made directly from the A matrix coefficients. We won't give the (quite complex) corresponding formulas here, as the same point can be addressed more easily with the Input Impedance Domain concept.

INPUT IMPEDANCE DOMAIN

The Input Impedance Domain (IID) of a linear filter is defined as

$$\mathcal{D} = \left\{ Z'_L(Z_L) \mid Z_L \in \mathbb{C}_+ \right\} \qquad \ldots(45)$$

where Z'_L is the input impedance of the filter, as given by

$$Z'_L = \frac{Z_L a_{22} - a_{12}}{a_{11} - Z_L a_{21}} \qquad \ldots(46)$$

The IID is either a half plane or a disc:

- if $a_{21} = 0$, then $a_{11}\ a_{22} = 1$, and $\mathcal{D}$ is obtained by translating $\mathbb{C}_+$ by the positive amount

$$-\frac{b_{12}}{b_{11}} \geq 0 \qquad \ldots(47)$$

in the direction of positive real part.

- if $a_{21} \neq 0$ and $a_{11} = 0$, then $\mathcal{D}$ is obtained by translating $\mathbb{C}_+$ by the positive amount

$$-\frac{c_{22}}{c_{21}} \geq 0 \qquad \ldots(48)$$

in the direction of positive real part.

- if $a_{21} \neq 0$ and $a_{11} \neq 0$, we define θ and φ such that

$$\theta \in [0, 2\pi[\ \text{ and } \ -a_{21}^2 = |a_{21}|^2 e^{j\theta} \qquad \ldots(49)$$

and

$$\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \ \text{ and } \ \frac{a_{11}}{a_{21}} = \left|\frac{a_{11}}{a_{21}}\right| e^{j\varphi} \qquad \ldots(50)$$

if $\varphi = \pm\frac{\pi}{2}$, then $\mathcal{D}$ is obtained by translating $\mathbb{C}_+$ by the positive amount

$$-\frac{c_{22}}{c_{21}} \geq 0 \qquad \ldots(51)$$

if $\varphi \neq \pm\frac{\pi}{2}$, then $\mathcal{D}$ is the disc of center

$$C = \frac{e^{-j\theta}}{2|a_{11}a_{21}\cos\varphi|} - \frac{a_{22}}{a_{21}} \qquad \ldots(52)$$

and radius

$$\rho = \frac{1}{2|a_{11}a_{21}\cos\varphi|} \qquad \ldots(53)$$

As we see, the input impedance domain may be plotted on a complex plane, whose real axis represents the input resistance, and the imaginary axis the input reactance. The value of mismatch impedance in the worst case at the output is obtained by considering the appropriate intersection of the IID with circles of constant mismatch attenuation.

A circle of constant mismatch attenuation $\mathcal{B}_T$ has a center

$$C'(\mathcal{B}_T) = \mathrm{Re}(Z_S)\left(2\mathcal{B}_T^2 - 1\right) - j\,\mathrm{Im}(Z_S) \qquad \ldots(54)$$

and the radius

$$\rho'(\mathcal{B}_T) = 2\,\mathrm{Re}(Z_S)\,\mathcal{B}_T\sqrt{\mathcal{B}_T^2 - 1} \qquad \ldots(55)$$

The value of the smallest $\mathcal{B}_T$ whose circle of constant mismatch attenuation intersects the IID, is the mismatch attenuation in the worst case at the output. This method of assessing this latter quantity is both simple and efficient.

LOSSLESS FILTERS

It can be shown that the IID of a lossless filter is $\mathbb{C}_+$. This implies

$$\mathcal{B}_0 = 1 \qquad \ldots(56)$$

and as the absorption attenuation is necessarily equal to 1, we obtain

$$\mathcal{A}_0 = 1 \qquad \ldots(57)$$

This last expression shows that lossless filters are of little help for power line filtering, where the worst case may in fact be realized.

EXAMPLE OF COMPUTATIONAL RESULTS

We investigated the behaviour of the filter shown in figure 2, known as [13] the series parallel dissipative low-pass filter.

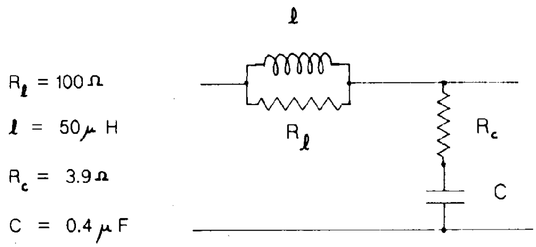

Figure 2 : The series parallel dissipative low-pass filter.

Our calculation were carried out with the FILTREXPERT package software, and some results are shown on figure 3 and 4.

Figure 5 shows a chart containing the circles described by (54) and (55). When the input impedance domain of figure 4 are drawn on this chart, determination of $\mathcal{B}_0(50\,\Omega)$ is obtained by looking at the intersections of the circles of the chart, and input impedance domain, as shown on figure 6.

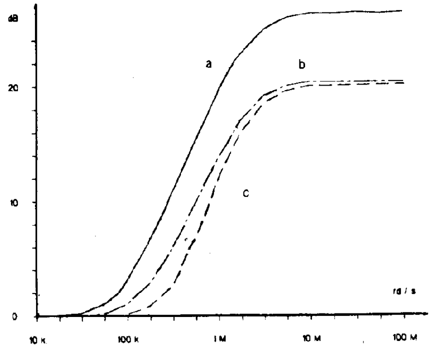

Figure 3 : 50 Ω system insertion loss (a), total attenuation in a worst case $\mathcal{A}_0(50\,\Omega)$ (b) and minimum attenuation (c) of the series-parallel dissipative low-pass filter.

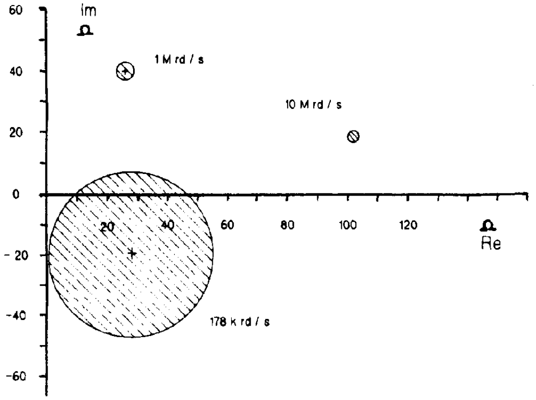

Figure 4 : Input impedance domain and mismatch attenuation of the series-parallel dissipative low-pass filter.

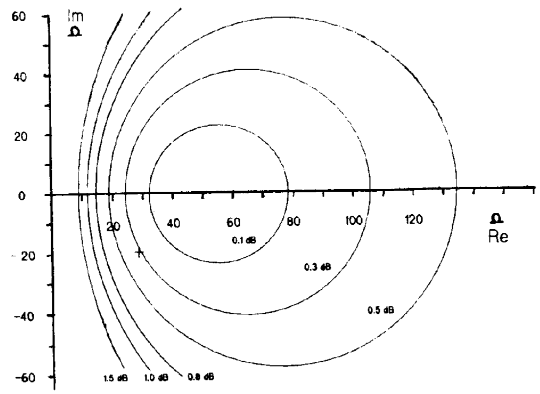

Figure 5 : Circles of constant $\mathcal{B}_o(50\Omega)$.

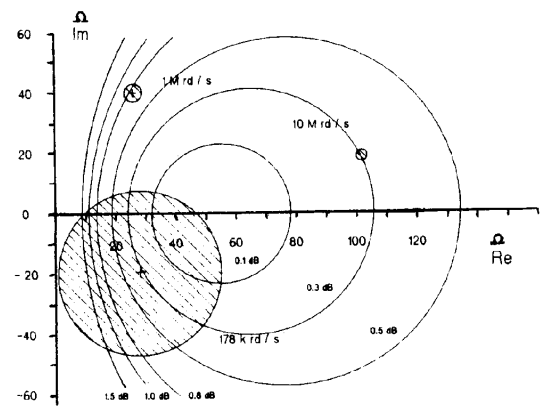

Figure 6 : Input impedance domain drawn on the chart of figure 5. One can read that
at 178 k rd/s $\mathcal{B}_o(50\Omega) \ll$ 0.1 dB,
at 1 M rd/s $\mathcal{B}_o(50\Omega) \simeq$ 1.0 dB,
at 10 M rd/s $\mathcal{B}_o(50\Omega) \simeq$ 0.3 dB.

In the present instance, the smallness of the mismatch attenuation in the worst case at the output explains the close values of total attenuation in the worst case at the output and minimum attenuation, as observed on figure 3.

DISCUSSION

We contend that the main concern of an EMC engineer is the maximum power at the output of a filter. For reduction of emission level, one has to make sure that the output power level of the power line filter does not exceed a certain value (it is clearly impossible to use voltages for this purpose). For reduction of susceptibility it is possible to use either the maximum power level or the maximum voltage at the output of the filter.

Design of power line filters should therefore be made with two worst-case theoretical tools: total attenuation in the worst case at the output (or at the input) and minimum voltage attenuation.

For the characterization of power-line filter the direct use of $\mathcal{A}$ or $\mathcal{B}$ present the following drawback: they are functions of an impedance, and therefore impractical to manipulate on data sheets.

On the other hand, it is possible, and easy, to use minimum attenuations and input impedance domain as descriptive tools, as they depend only on the filter itself, and can be plotted with frequency as variable. Once those two quantities are known it becomes possible to evaluate a pessimistic value for total attenuation in a worst case, as given by

$$\mathcal{A}_i(Z_L) \geq M \qquad \ldots(58)$$

$$\mathcal{A}_o(Z_S) \geq M \cdot \mathcal{B}_o(Z_S) \qquad \ldots(59)$$

We therefore recommend that minimum attenuation and input impedance domain be used for the characterization of power line filters.

CONCLUSION

Our hope is that designers and EMC engineers will become more and more aware of the specific aspects of power line filter theory, and particularly those related with the worst case behaviour of filters. The bases of this theory are not new, but we have presented here some improvements to it, in introducing the concepts of total attenuation in a worst case, of minimum attenuation, of mismatch attenuation in a worst case, and of input impedance domain.

Those complements to the theory, in addition to early works, offer the EMC engineer an alternative approach to conventional and incorrect design based solely on 50 Ohm insertion loss.

It has been stated that the sole use of 50 Ohm system insertion loss, though not rigorous, was nevertheless a good approximation of what would happen in the field: this belief is completely unfounded. This kind of calculation, even used with "generous" safety margins often leads to unsatisfactory performance.

Others contend that rigorous worst case theory is too complicated to be dealt with, and this opinion even appeared in a recent standard [17].

Our opinion and experience is that correct work is both feasible and economically worth doing.

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