Original text of the paper
"The Basis of a Theory for the Shielding by Cylindrical Generalized Screens"
with a few post-publication corrections in red

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3rd edition

This edition replaces the first and second editions dated February 24, 2001 and December 2, 2001. The following text, which is essentially identical with our revised paper, is intended to help the reader of the paper "The Basis of a Theory for the Shielding by Cylindrical Generalized Screens", published in the IEEE Transactions on EMC (November 2000 issue). Even though the IEEE did a nice job in publishing the paper, we felt necessary to straighten some differences with our work and editorial errors which appear in the paper:
— the lack of compliance of the published paper with our convention according to which all numbers, including the complex numbers (and quaternions) i and j, should not be written in italics like variables, makes some of the formula difficult to read (but they remain true),
— equations (30), (33), (34) and (36) of the published paper contain errors.

In the following text, all differences with our revised paper sent to the IEEE are printed in red (this convention does not apply to formulas).

Original text of the paper "The Basis of a Theory for the Shielding by Cylindrical Generalized Screens" with a few post-publication corrections in red — 3rd edition

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***Abstract* — Two new concepts applicable to cylindrical generalized screens are introduced: standard responses and standard excitations. The standard responses allow the description of any current and charge distribution on the screen. For a given generalized screen's cross-section, they can be derived from the complex potential of a simple electrostatic problem. The standard excitations are electromagnetic field configurations suitable for a description of fields created by sources external to the screen. All standard excitations are explicitely computed in the case of a circular cylindrical shield. We present only three standard excitations for the case of the elliptical cylinder, and for the case of a rectangular cylinder.**

I. INTRODUCTION

We have previously presented [1] the first results of an analysis of shielded multiconductor cables with respect to their shielding performances. This early work introduced the concept of five types of coupling between an external electromagnetic field and the cable. Our paper stated that the set of "type of coupling" considered was not complete, because an infinite series of type of coupling was necessary to describe the effect of charges on the the screen. We later decided [2] to refer to the five types of coupling defined initialy, as the five *main* types of coupling, because we *believed* they indeed gave an acceptable picture of the behaviour of most cables in many circumstances. However we were not able at that time to give a complete list of the coupling types and their associated parameters.

After our first article on this subject, we wrote several papers that improved our analysis, and also presented new experimental methods and results: a "parallel H-field probe" was designed and manufactured for the measurement of the parallel transfer impedance [2] [3] , and an "axial H-field probe" was built for the measurement of the axial transfer impedance [4] [5]. Interesting experimental results were also obtained with a rectangular TEM cell [1] and later with a GTEM cell [5].

Extensions of this work led us to a theory of the shielding performances of non-ideal cylindrical shields. The present paper is focused on two basic concepts of this theory, applicable to generalized screens: standard responses and standard excitations. These concepts are somewhat general because they allow a valid description of the behaviour of a shield, that can be implemented without restriction:
— on the nature of the excitation, any field structure being taken into account;
— on the shape of the shield, which is a cylinder of arbitrary cross-section;
— on what is inside the shield.

In this paper, a shield or screen denotes a structure of conductive material (electric or magnetic conductor) intended to reduce the penetration of electromagnetic fields into an assigned region. This structure is often very complex, for instance in the case of a braid or of a metallic tape wound around a multiconductor bundle. This is why we will often refer to a generalized screen containing the real screen.

A generalized screen or generalized shield (see[6], or § 10.2 of [7]) is defined as any combination of screens (made of electric conductor or magnetic conductor) and exclusion volumes, providing electromagnetic attenuation. Exclusion volumes are defined as volumes which may not contain field sources or conductors, potentially responsible for harmful coupling in the problem of interest. In practical computations, exclusion volumes are considered empty. In this paper the generalized screen will always be a closed and connected set, and its boundary will be the union of a cylindrical internal boundary and of a cylindrical external boundary, having no point in common, the latter surrounding the former. The internal boundary of the generalized shield surrounds a connected open set, later on referred to as the "volume inside the generalized screen". The points of space not included in the volume inside the external boundary, are said to be in the "volume outside the generalized screen", considered an open set. Thus, the volume inside the generalized screen, the volume of the generalized screen, and the volume outside the generalized screen cover all space and any two of them have an empty intersection.

II. Charges and Currents on the Generalized Screen

Throughout the paper, $\mathcal{B} = (\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ is a right-handed basis of orthogonal unit vectors. An origin O being chosen, the (rectangular) coordinates with respect to $\mathcal{B}$ are x, y and z, and the generalized screen's external boundary will be a cylinder $\mathcal{C}_E$ oriented along the Oz axis (see Fig. 1). The intersection $\Gamma(z_0)$ of this cylinder and a plane of equation $z = z_0$ in $\mathcal{B}$, is not necessarily circular, but it is a closed continuous curve. At any point on $\mathcal{C}_E$, we define a local right-handed basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_z)$ of orthogonal unit vectors: the unit-vector $\mathbf{e}_1$ is everywhere normal to $\mathcal{C}_E$ and pointing outward, and the unit-vector $\mathbf{e}_2$ is tangent to the surface and perpendicular to $\mathbf{e}_1$ and $\mathbf{e}_z$. The curvilinear coordinate u^2 on every $\Gamma(z_0)$ curves is chosen in such a way that it is dimensionless, that it is a bijective mapping from $]-\pi, \pi]$ to $\Gamma(z_0)$, and that the u^2 = constant curves on $\mathcal{C}_E$ are straight lines parallel to $\mathbf{e}_z$. (u^2, z) is therefore a system of orthogonal curvilinear coordinate on the cylinder $\mathcal{C}_E$. One can extend the definition of the coordinate u^2 to $\mathbb{R}$, so that the map that associates a coordinate u^2 to the corresponding point on $\Gamma(z_0)$ becomes a periodic function of period 2π.

We can obviously derive curvilinear coordinates (u^1, u^2, z) for the entire space, by properly choosing a family of cylinders $\mathcal{C}(u^1)$, one cylinder of the family being the external boundary $\mathcal{C}_E$ of the generalized screen and another matching the internal boundary $\mathcal{C}_I$ of the generalized screen, each cylinder of the family having at any point a local right-handed basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_z)$ of orthogonal unit vectors and a system of coordinate (u^2, z) built as above, and subject to the condition that the infinitesimal line element is:

$$\mathrm{d}\,\mathbf{r} = h_1 \mathbf{e}_1\, \mathrm{d}\, u^1 + h_2 \mathbf{e}_2\, \mathrm{d}\, u^2 + \mathbf{e}_z\, \mathrm{d}\, z \tag{1}$$

One can see that h_1 and h_2 are not dependent on z. Also, the external and internal boundaries of the generalized screen being cylinders of the family, they can respectively be described by the equations $u^1 = u^1_E$ and $u^1 = u^1_I$. The Fig. 1 illustrates our choice of coordinates, and shows a point M of curvilinear coordinates (u^1_M, u^2_M, z_M). One can build an infinity of different such curvilinear coordinate systems (for instance using problems of electrostatics for which u^2=constant surfaces are equipotentials).

In the first half of the following discussion, up to eq. (15), we shall only consider time-domain quantities.

If the generalized shield has a perfectly conducting external boundary, charges can only appear on this external boundary when all electromagnetic field sources are in the volume outside the generalized shield. Otherwise, charges may appear on the generalized shield's internal or external boundaries, and also inside the generalized shield (we may for instance have defined the generalized shield of a cable as made of a copper braid *and* a polyvinyl chloride jacket). The charge density ρ therefore depends on the three coordinates u^1, u^2, z, and we shall regard it as a distribution. An integration of the charge density ρ on the generalized shield thickness gives a quantity that has the same unit as a surface charge. However, it will be more convenient to introduce the "local per-unit-length charge density" (in C/m) that we will denote ρ_L, and define as:

$$\rho_L\left(u^2, z\right) = 2\pi \int_{u^1_I}^{u^1_E} \rho\, h_1 h_2 \, \mathrm{d}\, u^1 \tag{2}$$

where the path of integration is a portion of a u^2 = constant and z = constant line.

In the case of a perfectly conducting boundary at $u^1 = u^1_E$ and if all electromagnetic field sources are in the volume outside the generalized screen, charges are only present as a surface charge density ρ_S on the external boundary, and ρ is given by:

$$\rho\left(u^1,\, u^2,\, z\right) h_1 \,\mathrm{d}\, u^1 = \rho_S\left(u^2,\, z\right) \delta\left(u^1 - u^1_E\right) \mathrm{d}\, u^1 \tag{3}$$

where δ is the Dirac distribution. We therefore have in this special case:

$$\rho_S = \frac{\rho_L}{2\pi\ h_2} \tag{4}$$

Returning to the general case, we notice that, considered as a function of u^2, ρ_L is periodic of period 2π and can therefore be expanded in a Fourier series:

$$\rho_L\left(u^2, z\right) = \rho_{L\,0}(z) + \mathrm{Re}\left[\sum_{n=1}^{\infty} \rho_{L\,n}(z)\ \exp\left(\mathrm{i}\, n u^2\right)\right] \tag{5}$$

where the coefficient $\rho_{L\,0}$ is real, where for $n \geq 1$ the coefficient $\rho_{L\,n}$ is complex, and where $\mathrm{i}^2 = -1$ with Im(i) = 1.

It is important to notice that the total per-unit-length charge carried by the generalized screen is given by:

$$\iint_{\text{shield}} h_1 h_2\ \rho\ \mathrm{d}u^1\ \mathrm{d}u^2 = \frac{1}{2\pi} \int_0^{2\pi} \rho_L\ \mathrm{d}u^2 = \rho_{L\,0} \tag{6}$$

If the generalized shield has a perfectly conducting boundary, a surface current results. Otherwise, such a surface current can not take place, but an integration of the current density **j** regarded here as a distribution, on the generalized shield thickness (with respect to the variable u^1) gives a quantity that has the same unit as a surface current. However, it will be more convenient to introduce the "local current vector" (in A) that we will denote $\mathbf{i}_V$, and define as:

$$\mathbf{i}_V = i_{VR}\, \mathbf{e}_1 + j_{VO}\, \mathrm{h}_2\ \mathbf{e}_2 + i_{VA}\, \mathbf{e}_z \tag{7}$$

with

$$\begin{cases} i_{VR} = 2\pi \, h_1 \int_{u^1{}_I}^{u^1{}_E} h_2 \; \mathbf{j}.\mathbf{e}_1 \; \mathrm{d}\, u^1 \\ j_{VO} = 2\pi \int_{u^1{}_I}^{u^1{}_E} h_1 \; \mathbf{j}.\mathbf{e}_2 \; \mathrm{d}\, u^1 \\ i_{VA} = 2\pi \int_{u^1{}_I}^{u^1{}_E} h_1 h_2 \; \mathbf{j}.\mathbf{e}_z \; \mathrm{d}\, u^1 \end{cases} \tag{8}$$

where the path of integration is a portion of a u^2 = constant and z = constant line.

In the case of a perfectly conducting boundary at $u^1 = u^1{}_E$ and if all electromagnetic field sources are in the volume outside the generalized screen, there is a surface current $\mathbf{j}_S$ on the external boundary, and $\mathbf{j}$ is given by:

$$\mathbf{j}(u^1, u^2, z)\, h_1 \; \mathrm{d}\, u^1 = \mathbf{j}_S(u^2, z)\, \delta(u^1 - u^1_E)\, \mathrm{d}\, u^1 \tag{9}$$

where δ is the Dirac distribution. We therefore have in this particular case:

$$\mathbf{j}_S = \frac{\mathbf{i}_V}{2\pi \; h_2} \tag{10}$$

In this case and if the shield is isolated, $i_{VR} = 0$. This will be approximately the case for good shield, isolated. However i_{VR} should not be ignored for imperfect shields: the porpoising phenomenon (see [7], § 9.4.6) is precisely caused by such currents.

Returning to the general case, let us expand the two last coordinates of $\mathbf{i}_V$ in a Fourier series in the following manner:

$$j_{VO}(u^2, z) = j_{VO\,0}(z) + \mathrm{Re}\left[\sum_{n=1}^{\infty} j_{VO\,n}(z) \exp(\mathrm{i}\, n u^2)\right] \tag{11}$$

$$i_{VA}(u^2, z) = i_{VA\,0}(z) + \mathrm{Re}\left[\sum_{n=1}^{\infty} i_{VA\,n}(z) \exp(\mathrm{i}\, n u^2)\right] \tag{12}$$

where the coefficients $j_{VO\,0}$ and $i_{VA\,n}$ are real numbers, where for $n \geq 1$ the coefficients $j_{VO\,n}$ and $i_{VA\,n}$ are complex, and where $\mathrm{i}^2 = -1$ with Im(i) = 1.

We note that the total current flowing along the shield axis is:

$$\iint_{\text{shield}} h_1 h_2 \; \mathbf{j}.\mathbf{e}_z \; \mathrm{d}\, u^1 \; \mathrm{d}\, u^2 = \frac{1}{2\pi} \int_0^{2\pi} i_{VA} \; \mathrm{d}\, u^2 = i_{VA\,0} \tag{13}$$

We know that for a vector $\mathbf{F}$ of curvilinear coordinates (F_1, F_2, F_z) the value of the divergence is given by:

$$\nabla \cdot \mathbf{F} = \frac{1}{h_1 h_2}\left[\frac{\partial}{\partial u^1}(h_2 F_1) + \frac{\partial}{\partial u^2}(h_1 F_2) + \frac{\partial}{\partial z}(h_1 h_2 F_z)\right], \tag{14}$$

The equality div $\mathbf{j} + \partial\rho/\partial t = 0$ for the conservation of charges in time domain, once multiplied by $h_1 h_2$ and integrated with respect to u^1 along a u^2 = constant and z = constant path, becomes:

$$\int_{u^1_I}^{u^1_E}\left\{\frac{\partial}{\partial u^1}(h_2\,\mathbf{j}.\mathbf{e}_1)+\frac{\partial}{\partial u^2}(h_1\,\mathbf{j}.\mathbf{e}_2)+\frac{\partial}{\partial z}(h_1 h_2\,\mathbf{j}.\mathbf{e}_z)\right\}\mathrm{d}u^1+\frac{\partial}{\partial t}\int_{u^1_I}^{u^1_E}h_1 h_2\rho\,\mathrm{d}u^1=0 \qquad (15)$$

Assuming that the generalized shield is isolated, we have $\mathbf{j}.\mathbf{e}_1 = 0$ at $u^1 = u^1_{\mathrm{E}}$ and $u^1 = u^1_{\mathrm{I}}$, and suppressing a $e^{\mathrm{j}\omega t}$ dependance, we get in the frequency domain:

$$\frac{\partial i_{VA}}{\partial z}+\frac{\partial j_{VO}}{\partial u^2}+\mathrm{j}\,\omega\rho_L=0 \qquad (16)$$

where $\mathrm{j}^2 = -1$ with $\mathrm{Im}(\mathrm{j}) = 1$. We note that i used in (5) (11) and (12) corresponds to an angular parameter on the generalized screen, whereas j used in (16) corresponds to a phase parameter. Separately, they are the same complex number. However, because we are going to use both of them in expressions where they will have different meanings, it will be necessary to consider those two numbers, and also the phases of the complex quantities in equations where they appear, as having no relationship. Mathematically, the numbers i and j are in fact real quaternions, not complex numbers. They can be regarded as complex numbers only when: either only quaternions of the $\mathbb{R} + \mathrm{j}\mathbb{R}$ subspace are present in a formula, or only quaternions of the $\mathbb{R} + \mathrm{i}\mathbb{R}$ subspace are present in a formula. Any quantity related to i will be called "angular", and any quantity related to j will be called "frequential".

Let us also note that the choice of the definitions in (2), (7) and (8) was made for later convenience in (6), (13) and (16). This last expression can be easily expressed in the angular Fourier domain. Replacing i_{VA}, j_{VO} and ρ_{L} in (16) with (5), (11) and (12), then suppressing the $\exp(\mathrm{i}nu^2)$ dependence, we obtain:

$$\frac{\partial i_{VA\,0}}{\partial z}+\mathrm{j}\,\omega\rho_{L\,0}=0 \qquad (17)$$

and for any $n \in \mathbb{N}$, $n \geq 1$:

$$\frac{\partial i_{VA\,n}}{\partial z}+n\,j_{VO\,n}\,\mathrm{i}+\mathrm{j}\,\omega\rho_{L\,n}=0 \qquad (18)$$

(18) being an expression in the field of real quaternions. In (18), each of the three quantities $i_{\mathrm{VA\,n}}$, $j_{\mathrm{VO\,n}}$ and $\rho_{\mathrm{L\,n}}$ has a modulus, an angular phase as it is used in (5) or (11) or (12), and a frequential phase, as it is used in (16). We note that the second term of (18) does not show up in (17).

The results (17) and (18) deserve some more comments. As we know, it is experimentally possible on an electrically short length of generalized shield to have either negligible $\rho_{\mathrm{L\,0}}$ (this is what is obtained in a short-circuited triaxial set-up for the measurement of the per-unit-length transfer impedance of a coaxial cable). In this case (13) and (17) say that the conservation of charges becomes a conservation of the longitudinal current $i_{\mathrm{VA\,0}}$. For a current to flow on the generalized shield, it is therefore required that a return circuit exists, which provides a suitable path for the return current. Equation (18) says that a return circuit is not required for $n \geq 1$: a $j_{\mathrm{VO\,n}}$ per-unit-length current may become the necessary return current for $i_{\mathrm{VA\,n}}$ currents, as shown on Fig. 2 in the case $n = 1$, on an homogeneous metallic shield of strange cross section.

On Fig. 2, we observe a situation where the only non-negligible components of the local current $\mathbf{i}_{\mathrm{V}}$ flowing on the generalized screen, are $j_{\mathrm{VO\,1}}$ and $i_{\mathrm{VA\,1}}$. The local current, which only depends on u^2 and z is shown orthogonally projected on the generalized screen's external boundary. The values of the angular phases of $j_{\mathrm{VO\,1}}$ and $i_{\mathrm{VA\,1}}$ along the z axis, are such that vortex-like current appear. Such eddy current may show-up because of the local application (by external sources) of a varying magnetic field orthogonal to the Oz axis. On Fig. 2, at points A and C (lying in the same plane orthogonal to the axis, with the u^2

coordinate differing of about π) the $i_{VA\,1}$ components dominates. At points B and D the $j_{VO\,1}$ component dominates, and between these two points of equal u^2 coordinate, the angular phase difference of $j_{VO\,1}$ is about π.

The treatment of the generalized shield's behaviour for $n = 0$ is therefore a global problem, that is to say a problem involving the entire shield and a return circuit, but the shield behaviour *may be* a local problem for $n \geq 1$. A solution as presented on Fig. 2 for an homogeneous metallic shield is not always possible for complying with (18) on a given generalized screen: we can for instance think of a shield made of thin isolated wires parallel to the z axis. In this case, $j_{VO\,n}$ per-unit-length current cannot flow, and the establishment of $i_{VA\,1}$ currents is always a global problem.

III. Definition of the Types of Response

The shield being passive, the charge and currents on the shield may be regarded as a response to external stimuli. In this paragraph, we shall define the vocabulary and give some basic properties for a classification of charge and current distributions on the generalized screen. This vocabulary is centered on the word *response*.

Definition: Response. We shall call "response of the generalized screen", or simply "response" the pair of the current distribution and the charge distribution on the generalized screen.

Theorem 1. Let us denote the internal boundary by $\mathscr{C}_I$ and by $\mathscr{C}_E$ the external boundary of the generalized screen.
i) There exists a definition of the (u^1, u^2, z) coordinate system in the volume of the generalized screen and on the boundaries $\mathscr{C}_I$ and $\mathscr{C}_E$, such that:
— if over $\mathscr{C}_E$ we place a thin perfect electric conductor and charge it, a local per-unit-length charge density ρ_L independent of the variables u^2 and z would correspond to the electrostatic equilibrium of this conductor assumed the only object in space,
— if over $\mathscr{C}_I$ and $\mathscr{C}_E$ we place a thin perfect electric conductor, and obtain in this way the two electrodes of a capacitor filed with a medium of homogenous permittivity, the surfaces u^1 = constant in the volume of the generalized screen would be the equipotential surfaces when this capacitor is charged.
ii) Moreover, this definition of the u^2 coordinate in the volume of the generalized screen and on the boundaries $\mathscr{C}_I$ and $\mathscr{C}_E$ is unique but for an arbitrary additive constant.

proof: Let us assume that we place a per-unit-length charge Q, constant along the generalized screen, on the metallized external boundary $\mathscr{C}_E$, assumed alone in space. The surface $\mathscr{C}_E$ being invariant by any translation along Oz, the surface charge density ρ_S which appears on $\mathscr{C}_E$ is also invariant by such translations and does therefore not depend on z. Let us assume there exists a coordinate u^2 according to our whishes, and let us call L the length of the closed curve $\Gamma(0)$. Because the local per-unit-length charge density ρ_L is constant, it is equal to $\rho_{L\,0}$ and according to (6) we have:

$$Q = \frac{1}{2\pi}\int_0^{2\pi} \rho_L \; \mathrm{d}u^2 = \rho_L \tag{19}$$

using (4), we get:

$$h_2 = \frac{Q}{2\pi\rho_S} \tag{20}$$

If s is an arc length on $\Gamma(0)$, the coordinate u^2 satisfies:

$$\frac{\mathrm{d}\,u^2}{\mathrm{d}\,s} = \frac{2\pi\rho_S}{Q} \tag{21}$$

This differential equation uniquely defines u^2 but for an arbitrary additive constant, which establishes the unicity on $\mathscr{C}_E$. The unicity on the generalized screen and on the boundaries $\mathscr{C}_I$ is a direct consequence.

Let us now establish the existence of the coordinate system. The constant coordinate surfaces can be build easily. The only remaining problem is the mapping of the u^1 = constant and z = constant surfaces with u^2. The per-unit-length charge Q, being placed along the generalized screen, we see that if the surface charge density ρ_S which appears on $\mathscr{C}_E$ alone in space is integrable, (21) can be used to compute u^2. However ρ_S being defined as a derivative (derivative of a distribution) of the charge versus the coordinates z and s, it is necessarily Lebesgue-integrable. Also, it is also physically obvious that ρ_S is either everywhere positive or everywhere negative. We can therefore obtain u^2 from (21), and it will be monotonous. For a coordinate system built in this manner, according to (4) we have:

$$\rho_L = 2\pi h_2 \rho_S = Q \tag{22}$$

and ρ_S is a constant, QED.

Note that we will show in § VI that u^2 can often be computed with a conformal mapping, u^2 being therefore continuous.

Theorem 2: Let a definition of the (u^1, u^2, z) coordinate system satisfy the hypothesis of the theorem 1. The tangential component $i_{VA}\,\mathbf{e}_z + j_{VO}\,h_2\,\mathbf{e}_2$ of the local current vector $\mathbf{i}_V$ and the local per-unit-length charge density ρ_L are defined in a unique way (the proof is left to the reader).

Definition: Tangential response. Let a definition of the (u^1, u^2, z) coordinate system satisfy the hypothesis of the theorem 1. We call *tangential response* of the generalized screen the pair ($i_{VA}\,\mathbf{e}_z + j_{VO}\,h_2\,\mathbf{e}_2$, ρ_L) of the distribution of the tangential component $i_{VA}\,\mathbf{e}_z + j_{VO}\,h_2\,\mathbf{e}_2$ of the local current vector, and of the local per-unit-length charge density ρ_L.

Definition: Standard response. Let ($i_{VA}\,\mathbf{e}_z + j_{VO}\,h_2\,\mathbf{e}_2$, ρ_L) be a tangential response of the generalized screen. We define the standard responses of the generalized screen according the unique decomposition of the tangential response with (5), (11) and (12), in the following way:
i) a response of type $i_{VA\,0}$ is a $i_{VA}(u^2, z) = i_{VA\,0}(z)$ current;
ii) a response of type $\rho_{L\,0}$ is a $\rho_L(u^2, z) = \rho_{L\,0}(z)$ per-unit-length charge;
iii) a response of type $j_{VO\,0}$ is a $j_{VO}(u^2, z) = j_{VO\,0}(z)$ per-unit-length current;
iv) $\forall\, n \in \mathbb{N}^*$, a response of type $i_{VA\,n}$ is a $i_{VA}(u^2, z) = i_{VA\,n}(z)\exp(inu^2)$ current;
v) $\forall\, n \in \mathbb{N}^*$, a response of type $\rho_{L\,n}$ is a $\rho_L(u^2, z) = \rho_{L\,n}(z)\exp(inu^2)$ per-unit-length charge;
vi) $\forall\, n \in \mathbb{N}^*$, a response of type $j_{VO\,n}$ is a $j_{VO}(u^2, z) = j_{VO\,n}(z)\exp(inu^2)$ per-unit-length current.

Any standard response can also be regarded as a tangential response: $\forall\, n \in \mathbb{N}$,
— a response of type $i_{VA\,n}$ is the tangential response ($i_{VA\,n}(z)\exp(inu^2)\,\mathbf{e}_z$, 0);
— a response of type $\rho_{L\,n}$ is the tangential response (0, $\rho_{L\,n}(z)\exp(inu^2)$);
— a response of type $j_{VO\,n}$ is a tangential response ($j_{VO\,n}(z)\exp(inu^2)\,\mathbf{e}_2$, 0).

It shall be noted that this classification does not take into account possible i_{VR} currents. However, their mere existence is not denied nor neglected: they are only not described. This is of no consequence, because such current will usually be related to a response of one of the types defined above. As mentioned previously, the porpoising phenomenon on braided shields has for example been proved to be related to a response of type $i_{VA\,0}$.

Theorem 3: Any tangential response of a given generalized screen, can be written in the form of a sum of standard responses, each of a different type, and this expression is unique.

Proof: this theorem is simply the consequence of the existence and unicity of the contruction of ρ_L and $\mathbf{i}_V$, and of the Fourier series used in (5) (11) and (12).

Definition: Canonical decomposition. The unique expression defined in theorem 3 is called the canonical decomposition of the tangential response.

Definition: Pure tangential response. At a given point *z*, the tangential response is said "locally pure" if there is only one non-vanishing term in the canonical decomposition at this point. The tangential response is said "pure along the screen" if there is only one non-vanishing term in the canonical decomposition along the screen: in this case the tangential response of the entire screen is a single standard response.

Submitted to a given electromagnetic environment, the generalized shield will generally have a canonical decomposition containing the superposition of several standard responses. However, at this point we do not know if it is possible to create a pure response of a given type on a given shield, either locally or along the screen. Answering this question in detail is the purpose of the next paragraph.

IV. Independent Standard Responses

Definition: For a given generalized screen and for $p \in \mathbb{N}^*$, a *p*-tuple of standard responses is said physically independent if and only if, for any *p*-tuple of real quaternions, we can design a physically achievable experiment, in which the canonical decomposition of the tangential response along the screen will contain each of the standard responses multiplied by the quaternion of same index.

We note that this definition makes use of the trivial structure of vector space on the set of standard responses, regarded as tangential responses. Because of the orthogonality of the exponential functions in (5), (11) and (12), any *p*-tuple of standard responses of different types is linearly independent. Therefore a *p*-tuple of standard responses is linearly independent if and only if all standard response it contains are of different types.

Also, the physical independence of a *p*-tuple of standard responses implies the linear independence of the *p*-tuple. However the converse is obviously false, because the laws of physics *and the structure of the generalized screen* impose additional relations.

Theorem 4: A standard response of type $i_{VA\ 0}$ and a standard response of type $\rho_{L\ 0}$ are not physically independent, because they are related by (17), which implies that if the standard response of type $i_{VA\ 0}$ is known along the generalized screen, there is only one possible standard response of type $\rho_{L\ 0}$.

Theorem 4 means that the standard responses of type $\rho_{L\ 0}$ need not be taken into account if one intends to build the set of the physically achievable tangential responses along a generalized screen. The introduction of a standard response of type $\rho_{L\ 0}$ in the theory nevertheless addresses the need for a local description of the tangential response.

Theorem 5: $\forall\ n \in \mathbb{N}^*$, a standard response of type $i_{VA\ n}$, a standard response of type $j_{VO\ n}$ and a standard response of type $\rho_{L\ n}$ are not physically independent, because they are related by (18), which implies that if the standard responses of type $i_{VA\ n}$ and $\rho_{L\ n}$ are known along the generalized screen, there is only one possible standard response of type $j_{VO\ n}$.

Thus, for $n \geq 1$, the standard responses of type $j_{VO\ n}$ cannot exist independently of the other types of standard responses along the generalized screen. According to (18), the charges which appears because of a standard responses of type $j_{\mathrm{VO\,n}}$ and which are not removed by a longitudinal variation of $i_{\mathrm{VA\,n}}$ current of suitable amplitude, cause a per-unit-length charge $\rho_{\mathrm{L\,n}}$.

We may also state that for $n \geq 1$, the standard responses of type $j_{\mathrm{VO\,n}}$ need not be taken into account of one intends to build the set of the physically achievable tangential responses along the generalized screen. The introduction of a standard response of type $j_{\mathrm{VO\,n}}$ in the theory is however necessary for a local description of the tangential response.

Theorem 6: If the external boundary of a generalized screen is a perfect electric conductor, for $p \in \mathbb{N}^*$, any p-tuple of standard responses which do not include any standard response of type $\rho_{\mathrm{L\,0}}$, nor any standard response of type $j_{\mathrm{VO\,n}}$ for any $n \in \mathbb{N}^*$, this p-tuple containing a maximum of one standard response of each type, is physically independant.

Proof: Let us first place the generalized screen in vacuum. The external boundary being a perfect electric conductor, we have $i_{\mathrm{VR}} = 0$. We will only use a source in the volume outside the generalized screen. Therefore the only possible currents will be surface current on the external boudary. In this case (17) and (18) are equivalent to the conservation of charge.

From any linear combination of the p standard responses meeting the hypothesis of theorem 6, we can obviously create a new tangential response by adding standard responses of type $\rho_{\mathrm{L\,0}}$ and of type $j_{\mathrm{VO\,n}}$, in such a manner that (17) and (18) are satisfied. Let's call this tangential response the modified response. From the point of view of electromagnetism it is possible to move, with non-electromagnetic forces, the free charges of the conducting external boundary, in order to obtain the modified response on the generalized screen. These non-electromagnetic forces are usually taken into account with an electromotive force $\mathcal{E}$**.** Because we are only interested in surface current, we can postulate that $\mathcal{E}$ is tangential to the generalized screen's external boundary. If the screen was a medium of finite conductivity σ, and if a total electric field $\mathcal{E}$ was present, the effect of the non-electromagnetic force would be described by the equation:

$$\mathbf{j} = \sigma\left(\mathbf{E} + \mathcal{E}\right) \tag{23}$$

In the perfectly conducting medium of interrest here, we can only state that $\mathbf{E}+\mathcal{E}$ has a vanishing tangential component, which can be written, if we note $\mathbf{n}$ the unit vector normal to the generalized screen pointing outward:

$$\mathbf{E} = \left(\mathbf{E} \cdot \mathbf{n}\right)\mathbf{n} - \mathcal{E} \tag{24}$$

$\mathbf{E}$ being the electric field at the surface of the generalized screen.

The use of non-electromagnetic forces acting on the generalized screen can now be suppressed if we observe that their purpose is to compensate the force due to the tangential component of the electric field caused by the response. In other word, these non-electromagnetic forces were used to create a discontinuity of the tangential component of the electric field across the external boundary of the generalized screen. We know (cf. [8] p 34) that the same effect can be obtained with a surface density of magnetic current $\mathbf{M}_S$ placed on top of the generalized screen, taking on the value:

$$\mathbf{M}_S = \mathbf{E} \times \mathbf{n} = -\mathcal{E} \times \mathbf{n} \tag{25}$$

This surface density of magnetic current therefore allows the creation of the wanted modified response. One can show that this layer of magnetic surface current is equivalent to a double layer of electric surface current. It is clear that one could, with small enough conductors and generators, create a device approximating the double layer of surface current. This is what was meant by "a physically achievable experiment".

Theorem 7: If the external boundary of a generalized screen is a perfect electric conductor:
i) it is always possible to create a field configuration (i.e. field values as a function of space coordinates) that will produce a locally pure standard response of any given type,
ii) and possibilities of creating pure standard responses along the screen are only limited by the theorems 4 to 6 (the proof is left to the reader).

V. DEFINITIONS OF THE TYPES OF EXCITATION

Definition: Standard excitation. For a given generalized screen, we define a standard excitation at a point z, an electromagnetic field configuration produced by sources in the volume outside the generalized screen, which would produce, if the external boundary $\mathscr{C}_E$ was perfectly conducting, a locally pure standard response at z. The type of the standard excitation is by definition the type of this standard response.

We note that this definition is valid because we first established theorem 7. It introduces standard excitations of type $i_{VA\ n}$, of type $\rho_{L\ n}$ and of type $j_{VO\ n}$. It should be emphasized that many different electromagnetic field configurations are likely to produce the same standard excitation at a point z. There is therefore no uniqueness to be expected here.

A conjecture is that any electromagnetic environment (i.e. any applied electromagnetic field configuration in the volume outside the generalized shield) of a given generalized screen, can be written in the form of a sum of standard excitations at a point z, each of a different type. If true, this statement seems difficult to proove. We shall demonstrate it in § VII, in the case of generalized shield of circular cross-section.

VI. CALCULATION OF THE STANDARD RESPONSES AND EXCITATIONS

This paragraph will introduce the basics of a method for the computation of the standard responses and excitations. This method will be implemented in the § VII, § VIII and § IX, in three situations of increasing complexity for the shape of the external boundary: the cylinder of revolution, the elliptical cylinder, and the rectangular cylinder.

In fact this paragraph focuses on the main difficulty: defining a coordinate u^2 on the external boundary, according to the hypothesis of theorem 1. This being a two-dimensional potential distribution problem, it will be treated with analytic functions. In order to solve the problem of a charged conducting external boundary, only object in space, we shall consider a complex potential ζ which is an analytical function. The real part V of ζ will be the electric potential, and we shall note F the opposite of the imaginary part of ζ, usually refered to as the stream function.

We know (see [9] page 236) that the lines F = constante are field lines, and that the flux of the electric field (per unit length in the direction Oz) between the field lines $F = F_1$ and $F = F_2$ is simply $F_2 - F_1$. Specifically, on the conducting external boundary $\mathscr{C}_E$, the charge surface density ρ_S takes on the value:

$$\rho_S = \varepsilon_0 \frac{\mathrm{d}F}{h_2\ \mathrm{d}u^2} \tag{26}$$

In the case of interest, we assume that the local per-unit-length charge density ρ_L is constant for the charge distribution of electrostatic equilibrium. According to (20), we may write:

$$Q = 2\pi\, h_2 \rho_S = 2\pi\, \varepsilon_0 \frac{\mathrm{d}F}{\mathrm{d}u^2} \tag{27}$$

where Q is the charge on the generalized screen, per unit length along Oz. We can see that if the complex potential ζ is produced by the per-unit-length charge $Q = \pm\ 2\ \pi\epsilon_0 \times 1$ Volt, u^2 is defined by $u^2 = \pm\ F$ / 1 Volt, to which might be added any constant.

Once a complex potential is computed, u^2 is therefore known. The standard responses are then also known, because they are explicitely defined by (5) (11) and (12).

VII. Cylinder of Revolution as External Boundary

A. Standard Responses and Standard Excitations on the Circular Cylindrical Generalized Screen

Let us first closely examine the case of a generalized screen having an external boundary being a cylinder of revolution. If this external boundary was conducting, charged, and alone in space, it is well known (see [9] page 241) that a possible complex potential for the per-unit-length charge Q on the cylinder would be given[1] by:

$$\zeta = -\frac{Q}{2\pi\varepsilon_0} \ln(x + \mathrm{i}\, y) \tag{28}$$

so that we would obviously have:

$$\begin{cases} V = -\dfrac{Q}{2\pi\varepsilon_0} \ln\sqrt{x^2+y^2} \\ F = \dfrac{Q}{2\pi\varepsilon_0} \arg(x + \mathrm{i}\, y) \end{cases} \tag{29}$$

Thus, according to § VI, we can choose $u^2 = \theta$, θ being the argument of the complex variable, and take the coordinate (u^1, u^2, z) equal to the circular cylinder coordinate (r, θ, z), for which $h_1 = 1$ and $h_2 = r$. Because this separable coordinates are convenient for calculation, let's try to compute the standard excitations.

Let us therefore consider a (generalized) screen placed in vacuum, with a perfectly conducting circular cylindrical external boundary of radius r_0. In the volume outside the screen, suppressing a $e^{j\omega t}$ dependency, we may write the free-space (see [10], pp. 355-361, [8] pp. 198-204[2]), fields in circular cylindrical coordinates as:

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[1] In order not to confuse the reader, we shall not use z (already utilised) but x + iy for the complex variable.

[2] Both Stratton ([10], chap. VI, § 6.6, (28) and (29)) and Harrington ([8],chap. 5, § 5-1, (5-13) and (5-14)) use the same complex number for the angular phase and the frequential phase. This is erroneous because, for instance, a rotation of $\pi/2n$ is not equivalent to a time translation of $\pi/2\omega$. The appropriate expressions are (30), (31) and (32).

$$\begin{cases} E_r = \int_{-k}^{k} \mathrm{e}^{-\mathrm{j}hz} \left\{ -\mathrm{j}h \sum_{n=0}^{\infty} \left(a_n \frac{\partial \phi_n}{\partial r} + c_n \frac{\partial \psi_n}{\partial r} \right) - \mathrm{j}\eta_0 \frac{k}{r} \sum_{n=0}^{\infty} n \left(b_n \phi_n + d_n \psi_n \right) \mathrm{i} \right\} \mathrm{d}h \\ E_\theta = \int_{-k}^{k} \mathrm{e}^{-\mathrm{j}hz} \left\{ -\mathrm{j} \frac{h}{r} \sum_{n=0}^{\infty} n \left(a_n \phi_n + c_n \psi_n \right) \mathrm{i} + \mathrm{j} k \eta_0 \sum_{n=0}^{\infty} \left(b_n \frac{\partial \phi_n}{\partial r} + d_n \frac{\partial \psi_n}{\partial r} \right) \right\} \mathrm{d}h \\ E_z = \int_{-k}^{k} \mathrm{e}^{-\mathrm{j}hz} \left(k^2 - h^2 \right) \sum_{n=0}^{\infty} \left(a_n \phi_n + c_n \psi_n \right) \mathrm{d}h \\ H_r = \int_{-k}^{k} \mathrm{e}^{-\mathrm{j}hz} \left\{ \mathrm{j} \frac{k}{\eta_0 r} \sum_{n=0}^{\infty} n \left(a_n \phi_n + c_n \psi_n \right) \mathrm{i} - \mathrm{j}h \sum_{n=0}^{\infty} \left(b_n \frac{\partial \phi_n}{\partial r} + d_n \frac{\partial \psi_n}{\partial r} \right) \right\} \mathrm{d}h \\ H_\theta = \int_{-k}^{k} \mathrm{e}^{-\mathrm{j}hz} \left\{ \frac{-\mathrm{j}k}{\eta_0} \sum_{n=0}^{\infty} \left(a_n \frac{\partial \phi_n}{\partial r} + c_n \frac{\partial \psi_n}{\partial r} \right) - \mathrm{j} \frac{h}{r} \sum_{n=0}^{\infty} n \left(b_n \phi_n + d_n \psi_n \right) \mathrm{i} \right\} \mathrm{d}h \\ H_z = \int_{-k}^{k} \mathrm{e}^{-\mathrm{j}hz} \left(k^2 - h^2 \right) \sum_{n=0}^{\infty} \left(b_n \phi_n + d_n \psi_n \right) \mathrm{d}h \end{cases} \tag{30}$$

where: the integer n and the real propagation "constant" h are separation "constants",
a_n and c_n are (h dependent) amplitude distributions, expressed in Vm^2,
b_n and d_n are (h dependent) amplitude distributions, expressed in Am^2,
η_0 is the free-space wave impedance,
k is the wave number ω/c_0,

and where the functions ϕ_n and ψ_n are defined by the equations stemming from the separation of variables. In the case where $h \neq \pm k$, this is a Bessel differential equation and we obtain:

$$\begin{cases} \phi_n(r) = \mathrm{H}_n^{(1)}\left(\sqrt{k^2 - h^2}\, r \right) \mathrm{e}^{\mathrm{i}n\theta} \\ \psi_n(r) = \mathrm{H}_n^{(2)}\left(\sqrt{k^2 - h^2}\, r \right) \mathrm{e}^{\mathrm{i}n\theta} \end{cases} \tag{31}$$

which are functions depending on h and where $\mathrm{H}_n^{(1)}$ and $\mathrm{H}_n^{(2)}$ are Hankel functions with a frequential phase. In the case $h = \pm k$, there is no Bessel equation, and the differential equation leads us to:

$$\begin{cases} \phi_n(r) = \begin{cases} \mathrm{e}^{\mathrm{i}n\theta} \left(r/r_0 \right)^n & \text{for } n \neq 0 \\ \mathrm{e}^{\mathrm{i}n\theta} & \text{for } n = 0 \end{cases} \\ \psi_n(r) = \begin{cases} \mathrm{e}^{\mathrm{i}n\theta} \left(r/r_0 \right)^{-n} & \text{for } n \neq 0 \\ \mathrm{e}^{\mathrm{i}n\theta} \ln\left(r/r_0 \right) & \text{for } n = 0 \end{cases} \end{cases} \tag{32}$$

For $h \neq \pm k$, we note that the functions ϕ_n are cylindrical waves propagating toward the Oz axis, and that the functions ψ_n are cylindrical waves propagating from the Oz axis, so that only the fields components related to the ψ_n functions can be caused by the current and charges on the screen.

Because there are angular phase and frequential phase dependancies, we note that the amplitude distributions a_n, c_n, b_n and d_n, as well as the field amplitude E_r, E_θ, E_z, H_r, H_θ and H_z are Hamilton's quaternions.

For $h = \pm k$ we can check that a_0 and b_0 are not associated with any field component. For $h = \pm k$ and $n \neq 0$ the electromagnetic fields depend only on the variables u_n and v_n defined by:

$$\begin{cases} u_n\, \delta(h-k) = a_n \pm \eta_0 b_n\, \mathrm{i} \\ v_n\, \delta(h-k) = c_n \mp \eta_0 d_n\, \mathrm{i} \end{cases} \tag{33}$$

where δ is the Dirac distribution. The variables u_n and v_n therefore have the dimension of Vm. In this case we in fact have a TEM wave propagating along the shield axis, and the only non vanishing field components are:

$$\begin{cases} E_r = \mp \dfrac{\mathrm{j}kn}{r}\, \mathrm{e}^{\mp \mathrm{j}kz}\left(u_n (r/r_0)^n - v_n (r/r_0)^{-n}\right) \mathrm{e}^{\mathrm{i}n\theta} \\ E_\theta = \mp \dfrac{\mathrm{j}kn}{r}\, \mathrm{e}^{\mp \mathrm{j}kz}\left(u_n (r/r_0)^n + v_n (r/r_0)^{-n}\right) \mathrm{i}\, \mathrm{e}^{\mathrm{i}n\theta} \\ H_r = \mp \dfrac{1}{\eta_0} E_\theta \\ H_\theta = \pm \dfrac{1}{\eta_0} E_r \end{cases} \tag{34}$$

We can observe that we only considered the values of h giving rise to periodic solutions along the screen axis, which correspond to h real, included in the interval $[-k, k]$, whence our integration path in (33).

The boundary conditions on the (perfect) shield's external boundary is $E_\theta = E_z = H_r = 0$ at $r = r_0$.

For $h \neq \pm k$, the boundary condition is equivalent to:

$$\begin{cases} \forall n \quad a_n\, \mathrm{H}_n^{(1)}\left(\sqrt{k^2 - h^2}\, r_0\right) + c_n\, \mathrm{H}_n^{(2)}\left(\sqrt{k^2 - h^2}\, r_0\right) = 0 \\ \forall n \quad b_n \dfrac{\mathrm{d}\mathrm{H}_n^{(1)}}{\mathrm{d}x}\left(\sqrt{k^2 - h^2}\, r_0\right) + d_n \dfrac{\mathrm{d}\mathrm{H}_n^{(2)}}{\mathrm{d}x}\left(\sqrt{k^2 - h^2}\, r_0\right) = 0 \end{cases} \tag{35}$$

so that, the variable h not withstanding, the fields depend on two arbitrary amplitudes.

For $h = \pm k$, the boundary condition is equivalent to:

$$\begin{cases} \forall n \neq 0 \qquad (a_n \pm \eta_0 b_n\, \mathrm{i}) + (c_n \mp \eta_0 d_n\, \mathrm{i}) = 0 \\ d_0 = 0 \end{cases} \tag{36}$$

so that taking (33) into account, the sign of $h = \pm k$ not withstanding, the fields depend only one arbitrary amplitude.

For any field configuration, from (8) and boundary conditions on the external boundary, we find:

$$\begin{cases} \rho_L = 2\pi r_0\, \varepsilon_0\; E_r \\ j_{VO} = -2\pi\; H_r \\ i_{VA} = 2\pi r_0\; H_\theta \end{cases} \tag{37}$$

At this stage, we can see that the components of H_θ of index n correspond to standard excitations of type $i_{\mathrm{VA\,n}}$, that the components of H_z of index n correspond to standard excitations of type $j_{\mathrm{VO\,n}}$, and that the components of E_r of index n correspond to standard excitations of type $\rho_{\mathrm{L\,n}}$.

B. Simple Combinations of Standard Excitations on the Circular Cylindrical Generalized Screen

It is now possible to establish the complete list of the free-space field configurations which lead to the simplest combinations of standard excitations, allowed by theorems 4 and 5 along any generalized screen with a circular cylindrical external boundary:
– A standard excitation of type $i_{\mathrm{VA\,0}}$, only combined with a standard excitation of type $\rho_{\mathrm{L\,0}}$ as prescribed by theorem 4 can be created along the generalized screen: either with an electromagnetic field including only $h = \pm k$ components with all amplitude distributions equal to zero except c_0, or with an electromagnetic field including only $h \neq \pm k$ components with all amplitude distributions equal to zero except a_0 and c_0 related by (35). We note that in the special case $h = 0$, the canonical decomposition only contains the standard excitation of type $i_{\mathrm{VA\,0}}$.
– A pure standard excitation of type $j_{\mathrm{VO\,0}}$ can be created along the generalized screen, with an electromagnetic field including only $h \neq \pm k$ components with all amplitude distributions equal to zero except b_0 and d_0 related by (35).
– For $n \geq 1$, a standard excitation of type $i_{\mathrm{VA\,n}}$, only combined with a standard excitation of type $\rho_{\mathrm{L\,n}}$ as prescribed by theorem 5 can be created along the generalized screen: either with an electromagnetic field including only $h = \pm k$ components with the amplitude distributions u_n and v_n related by (36), or with an electromagnetic field including only $h \neq \pm k$ components with all amplitude distributions equal to zero except a_n and c_n related by (35).
– For $n \geq 1$, a standard excitation of type $j_{\mathrm{VO\,n}}$, only combined with a standard excitation of type $\rho_{\mathrm{L\,n}}$ as prescribed by theorem 5 can be created along the generalized screen, with an electromagnetic field including only $h \neq \pm k$ components with the amplitude distributions a_n , c_n , b_n and d_n related by (35) and the additional relation cancelling H_θ.
– For $n \geq 1$, a standard excitation of type $j_{\mathrm{VO\,n}}$, only combined with a standard excitation of type $i_{\mathrm{VA\,n}}$ as prescribed by theorem 5 can be created along the generalized screen, with an electromagnetic field including only $h \neq \pm k$ components with the amplitude distributions a_n , c_n , b_n and d_n related by (35) and the additional relation cancelling E_r, this being in general only possible for $h \neq 0$.

C. Locally Pure Excitations on the Circular Cylindrical Generalized Screen

We now understand that, with an appropriate choice of incidence, or by taking advantage of interference, it is possible to locally create a field configuration where only one standard excitation dominates. Thus:
– In order to obtain a standard excitation of type $i_{\mathrm{VA\,0}}$ locally pure in the neighbourhood of $z = 0$, we can for instance combine a field component $h = k$ and a field component $h = -k$, both of them with the same amplitude c_0. This is easily obtained in the laboratory, on a fraction of wavelength along the 0z axis in a short-circuited triaxial test fixture. This implementation is much simpler than the creation of a cylindrical wave with $h = 0$.
– In order to obtain a standard excitation of type $\rho_{\mathrm{L\,0}}$ locally pure in the neighbourhood of $z = 0$, we can for instance combine a field component $h = k$ and a field component $h = -k$, both of them with their amplitudes c_0 of opposite value. This is easily obtained in the laboratory, on a fraction of wavelength along the 0z axis in an open-circuited triaxial (or quadraxial) test fixture.
– Obtaining locally a pure standard excitation of $j_{\mathrm{VO\,0}}$ is easy, as we have previously seen. We can for instance create an electromagnetic field having only a $h = 0$ component with all amplitude distributions equal to zero except b_0 et d_0 related by (35). This is easily (but approximately) obtained in the laboratory provided r_0 is much smaller than the wavelength, if the screen is installed on the axis of a solenoïd each turn of which would be separately excited by a current source in phase with the others.
– For $n \geq 1$, we can for instance obtain a standard excitation of type $i_{\mathrm{VA\,n}}$ locally pure in the

neighbourhood of $z = 0$, by combining a field component $h = k$ and a field component $h = -k$, with equal amplitudes u_n and v_n, these amplitudes being related by (36). This is possible to obtain in the laboratory on a fraction of wavelength along the 0*z* axis in a test set made of 2*n* short-circuited tapes connected to a symetrical generator.
– For $n \geq 1$, we can for instance obtain a standard excitation of type $\rho_{L\,n}$ locally pure in the neighbourhood of $z = 0$, by combining a field component $h = k$ and a field component $h = -k$, with opposite amplitudes u_n and v_n, these amplitudes being related by (36). This is possible to obtain in the laboratory on a fraction of wavelength along the 0*z* axis in a test set made of 2*n* open-circuited tapes connected to a symetrical generator
– For $n \geq 1$, we need at least three different incidences to obtain a standard excitation of type $j_{VO\,n}$ locally pure in the neighbourhood of $z = 0$. We will for instance combine a field component $h = 0$, the amplitudes a_n , c_n , b_n and d_n of which will be related by (35) and the additional relation cancelling H_θ, with a field component $h = k$ and a field component $h = -k$, with opposite amplitudes u_n and v_n, these amplitudes being related by (36) and taking on an appropriate value for cancelling at $z = 0$ the E_r part of the component $h = 0$.

Note 1: In practice, the locally pure standard excitations are only easily implemented in the laboratory for a generalized screen sample with transverse dimensions much smaller than the area where the excitation is locally pure, which is itself necessarily much smaller than the wavelength, except for j_{VO0}. For $j_{VO\,0}$ though, it is also much easier to implement the pure standard excitation on a sample of electrically small cross-section.

Note 2: For $n \geq 1$ if we create a standard excitation of type $j_{VO\,n}$ locally pure in the neighbourhood of $z = 0$, we note that as arround z = 0 we necesserily find the currents which have been introduced to remove the charges brought by the $j_{VO\,n}$ response. If the radius r_0 if small compared to the wavelength, (7) and (18) show that the current along $\mathbf{e}_2$ will only dominate the current along $\mathbf{e}_z$ on a distance of the order of r_0/n. We should therefore question the possibility of making useful laboratory measurements in this situation.

Note 3: In a test set-up with $n \geq 1$ short-circuited or open-circuited tapes meant to obtain standard excitations standard of type $i_{VA\,n}$ or of type $\rho_{L\,n}$ locally pure in the neighbourhood of $z = 0$, the shape of the different tape should be as close as possible to the equipotential surfaces of the transverse electrostatic problem for TEM propagation.

VIII. Elliptical Cylinder as External Boundary

Let's now study the case of a generalized screen with an external boundary having the shape of an elliptical cylinder. We shall note *a* the semi-major axis and *b* the semi-minor axis of the ellipse. For an appropriate choice of the orientation of O*x*, a parametric equation of the external boundary can be written:

$$\begin{cases} x = a\cos t \\ y = b\sin t \end{cases} \tag{38}$$

It is well known (see [9] page 242) that if the elliptical cylinder is charged with a per-unit-length charge *Q* and alone in space, his complex potential is given by:

$$\zeta = \frac{-Q}{2\pi\varepsilon_0}\operatorname{arccosh}\left(\frac{x + \mathrm{i}\,y}{\sqrt{a^2 + b^2}}\right) \tag{39}$$

In a plane orthogonal to the axis, the rectangular coordinates of a point can be deduced from the values of the potential and stream functions:

$$\begin{cases} x = \sqrt{a^2+b^2}\,\cosh\dfrac{2\pi\varepsilon_0}{Q}V\,\cos\dfrac{2\pi\varepsilon_0}{Q}F \\ y = -\sqrt{a^2+b^2}\,\sinh\dfrac{2\pi\varepsilon_0}{Q}V\,\sin\dfrac{2\pi\varepsilon_0}{Q}F \end{cases} \tag{40}$$

The equipotential lines and field lines can be directly obtained from these two formulas. They are respectively confocal ellipses and hyperbolaes.

From the § VI and the comparison of (38) and (40), we can see that on the screen external boundary, we can take $u^2 = t$. More generally, in the volume outside the generalized screen and the external boundary we can take the coordinates (u^1, u^2, z) simply equal to (ξ_1, ξ_2, z) where ξ_1 and ξ_2 are the elliptic coordinates, which are respectively equal to V and −F given by (40) for $Q = -2\,\pi\epsilon_0 \times 1$ Volt.

From (41) we can compute the complex field given by:

$$E_x - \mathrm{i}\,E_y = -\frac{\mathrm{d}\,\zeta}{\mathrm{d}\,z} \tag{41}$$

The result is:

$$E_x - \mathrm{i}\,E_y = \frac{Q}{2\pi\varepsilon_0}\,\frac{1}{\sqrt{(x+\mathrm{j}\,y)^2 - (a^2+b^2)}} \tag{42}$$

With (38) and (42), we easily establish that, on the external boundary of the generalized screen, the surface charge density is:

$$\rho_S = \frac{Q}{2\pi\,ab}\,\frac{1}{\sqrt{\dfrac{x^2}{a^4}+\dfrac{y^2}{b^4}}} \tag{43}$$

The value of h_2 on the boundary can be directly obtained from (40), and we find:

$$h_2 = \sqrt{a^2\sin^2 t + b^2\cos^2 t} \tag{44}$$

Note that the use of (4), with $\rho_L = Q$ from (6), and (44), allows one to establish (43) without using the electric field normal to the external boundary.

The problem of the elliptical cylinder being more complex than that of the circular cylinder, we will not try to compute all standard excitations. In fact, we will limit ourself to defining possible laboratory set-up for the creation of the standard excitations of types $i_{\mathrm{VA}\,0}$, $\rho_{\mathrm{L}\,0}$, and $j_{\mathrm{VO}\,0}$, because they are always trivial:

– A standard excitation of type $\rho_{\mathrm{L}\,0}$ obviously corresponds to the natural electrostatic charge distribution of the charged generalized screen (assumed with a conducting external boundary) in free space, for which we can compute the equipotential surfaces. The metallization of one of these surfaces with a good conductor, this surface being choosen close enough from the screen for the propagation to be limited to the TEM mode at the frequencies of interest, is a natural way of constructing a test set-up for producing type $\rho_{\mathrm{L}\,0}$ standard excitation. The standard excitation will be locally pure provided the instrument is used open-circuited, as in the case of the circular cylinder.

– A standard excitation of type $i_{\mathrm{VA}\,0}$ corresponds to the natural distribution of surface currents on the generalized screen (the external boundary of which is assumed perfectly conducting) when the instrument

defined for the production of standard excitation of type $\rho_{L\,0}$ is used short-circuited. This is because, for the TEM propagation mode in a lossless wave-guide, the transverse current distribution is equal to the charge distributions multiplied by a constant (see [11], p. 248).
– A standard excitation of type $j_{VO\,0}$ is easily obtained in the laboratory provided the cross-section of the generalized screen is much smaller than the wavelength, if the screen is installed on the axis of a solenoïd each turn of which would be separately excited by a current source in phase with the others, as with the circular cylindrical shield.

IX. Rectangular Cylinder as External Boundary

This paragraph will discuss the case of a generalized screen for which the external boundary is a cylinder of rectangular cross-section. We shall note a and b respectively the larger and the smaller dimensions of the rectangle. The equation of the rectangle shall be:

$$\begin{cases} y=0 \text{ or } y=-b & \text{for } \; x\in\left[-\dfrac{a}{2}, \dfrac{a}{2}\right] \\ x=-\dfrac{a}{2} \text{ or } x=\dfrac{a}{2} & \text{for } \; y\in[-b, 0] \end{cases} \tag{45}$$

We shall use the known complex potential of a thin wire at $X = 0$ $Y = 1$, carrying a per-unit-length charge Q, installed above an infinite ground plane $Y = 0$ (see [12], page 209):

$$\zeta = \frac{Q}{2\pi\varepsilon_0}\ln\left(\frac{X+\mathrm{i}(Y+1)}{X+\mathrm{i}(Y-1)}\right) \tag{46}$$

We now transform this problem into the problem of the perfectly conducting external boundary of the rectangular cylinder charged in free space, with an appropriate Schwarz-Christoffel transform (see [9], page 314[3]):

$$x+\mathrm{i}\,y = \frac{A}{k}\int_0^{X+\mathrm{i}Y} \frac{\sqrt{(h^2-t^2)(1-k^2t^2)}}{(t^2+1)^2}\,\mathrm{d}t \tag{47}$$

where h and k are defined as the solutions of :

$$\begin{cases} \displaystyle\int_h^{\frac{1}{k}} \frac{\sqrt{(t^2-h^2)(1-k^2t^2)}}{(t^2+1)^2}\,\mathrm{d}t = \frac{2b}{a}\int_0^h \frac{\sqrt{(h^2-k^2)(1-k^2t^2)}}{(t^2+1)^2}\,\mathrm{d}t \\ \displaystyle\int_{\frac{1}{k}}^{\infty} \frac{\sqrt{(t^2-h^2)(k^2t^2-1)}}{(t^2+1)^2}\,\mathrm{d}t = \int_0^h \frac{\sqrt{(h^2-k^2)(1-k^2t^2)}}{(t^2+1)^2}\,\mathrm{d}t \end{cases} \tag{48}$$

and A by:

$$A = \frac{b}{\displaystyle\int_h^{\frac{1}{k}} \frac{\sqrt{(t^2-h^2)(1-k^2t^2)}}{(t^2+1)^2}\,\mathrm{d}t} \tag{49}$$

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[3] The formulas (365), (366) and (367) of [9] should be modified, because one cannot arbitrarily assume $h = 1$ for the computation of the potential external to a rectangular cylindrical boundary.

This conformal mapping transforms the real axis $Y = 0$ into the rectangle defined by (45), the open half-plane $Y > 0$ into the surface outside this rectangle, and the point $X = 0$, $Y = 1$ into infinity. Let us note that the real part V of the complex potential vanishes on the external boundary of the screen.

Eq. (48) and (49) can be solved numerically without difficulty. The formulas (46) and (47) then become simple and effective means for computing the complex potential at any point in the volume outside the screen. Specifically, on the external boundary of the screen, we have $Y = 0$, and (47) gives:

$$F = \frac{-Q}{\pi \varepsilon_0} \arg(X + \mathrm{i}) \tag{50}$$

Using (27) and choosing $u^2 = F + \pi$ for $Q = 2\pi\epsilon_0$ we get:

$$u^2 = \pi - 2\arctan\left(\frac{1}{X}\right) \tag{51}$$

on the external boundary of the screen, with a cut at $X = 0$. Note that for $X \geq 0$ we have:

$$u^2 = 2\arctan X \tag{52}$$

on the external boundary of the screen. The formulas (47) and (51) allow to compute effectively the position of any point on the external boundary of the screen for a given u^2 coordinate, and afterwards to extend this definition of u^2 to the volume outside the screen. This is what we have done on Fig. 3 in the case $a = 2b$, after plotting the external boundary (curve $V = 0$), we plotted some u^2 = constant curves. The latter were computed by integration of (47) along the field lines derived from:

$$X + \mathrm{i}\,Y = \mathrm{i}\,\frac{\exp\left(V + \mathrm{i}\left[\pi - u^2\right]\right) + 1}{\exp\left(V + \mathrm{i}\left[\pi - u^2\right]\right) - 1} \tag{53}$$

with u^2 constant and V as variable, taking on the value 0 on the external boundary of the screen.

This investigation shows that the coordinate u^2 can be easily defined for an external boundary having angles, without having to resort to numerical methods for a two-dimensional problem. It also shows that even in this case, u^2 is continuous.

As in the case of the elliptical cylinder, we can establish without additional computation the definition of a possible laboratory set-up for the creation of the standard excitations of the types $i_{\mathrm{VA}\,0}$, of type $\rho_{\mathrm{L}\,0}$, and of type $j_{\mathrm{VO}\,0}$. The definition is exactly identical to that presented for elliptical cylinder, only the shape of the equipotential surfaces being different.

X. Conclusion

We have presented in this paper the first part of a theory of the screening properties of cylindrical generalized shields. We have only defined a classification of the responses and excitations in the presence of sources in the volume outside the generalized screen. This classification is based on the definitions and basic properties of the standard response and standard excitations, in curvilinear coordinates.

It should be noticed that this theoretical presentation is not limited in the frequency domain. Even though it extensively uses the property of cylindrical shields with a perfectly conducting external boundary, this theory is applicable to any cylindrical screen, at any frequency. It is therefore of interest for the study of screened cables, shielded conduits, and other long conducting structures like fuselages.

Establishing the standard reponses on a given screen was shown to be equivalent to finding a u^2 cooordinate meeting some requirements. We have shown on three different examples how this coordinate could be computed with analytical functions.

In the case of an external boundary of revolution we could define test set-up for locally producing any pure standard excitation. This work could be done for boundaries leading to other separable coordinates.

We note that for an arbitrary shape of the external boundary of the generalized screen, generating locally pure standard excitations of the types $i_{VA\,0}$, of type $\rho_{L\,0}$, and of type $j_{VO\,0}$ is always trivial. However in general, it will not be possible to determine analytically the possible sources for other pure standard excitations, as we have done for the circular cylinder. For this problem, the analytical results can probably not be obtained much further than formula (25), which in practice requires the computation of the electric field produced by the standard response.

In the case of a perfectly conducting (electric) external boundary, the present paper also discussed the combination of standard responses caused by the most general external field distribution.

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