Characterization of a Cylindrical Screen for External Excitations and Application to Shielded Cables

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***Abstract*—The main purpose of this paper is to present the definition of sets of parameters, which provide an intrinsic characterization of the shielding performances of a cylindrical generalized screen submitted to an external field. As examples, we detail two implementations of this approach at the end of the paper, for the computation of the voltages induced by an incident field on a short section of a multiconductor shielded cable. More precisely, our approach is based on an expansion of the fields which may excite the generalized screen, into a combination of "standard excitations." They are defined in such a way that the shielding performances for a given standard excitation can be characterized in a simple manner, for instance, a single scalar parameter. In the case of shielded cables of electrically small cross section, this leads us to the rigorous introduction of the parameters for the "five main types of coupling," for which some experimental results have already been published. We then establish the exact induced current and voltage on a section of cable running above a ground plane, when an incident field propagates parallel to the ground plane, in the case of a longitudinal excitation, and in the case of a transverse excitation. We use these results to provide the value of the voltages at each end of a short section of cable characterized with the parameters for the five main types of coupling.**

***Index Terms*—Axial transfer impedance, cable, cylindrical screen, electromagnetic (EM) coupling, parallel transfer admittance, parallel transfer impedance, per-unit-length transfer impedance, quaternions, radial electric coupling coefficient, shielded multiconductor cable, shielding, shielding matrix, standard excitation, standard response.**

I. Introduction

It is well known that the per-unit-length transfer impedance is a suitable parameter for characterizing the shielding performances of a coaxial cable with a homogenous screen. This characterization is known to be intrinsic to the cable (i.e., it describes only the coaxial cable, not the cable behavior in a given measurement setup) and it can be used to compute the induced signal in any configuration of the cable. This paper addresses the similar (but more general) question of the characterization of the shielding performances of a cylindrical generalized screen, for instance, the shield of a multiconductor shielded cable of noncircular cross section. We will consider that the generalized screen is submitted to an external field. The behavior of the screen in this external field being complicated, it is generally not possible to find a simple parameter to characterize the shielding performance of the screen in any external field. We will therefore use a divide and conquer approach. We will implement some sort of expansion of the applied field into a combination of "standard excitations," so chosen that we can use a small number of parameters (ideally a single scalar) for the characterization of the generalized screen for each standard excitation.

In an earlier paper presenting the theoretical basis of our approach to the modeling of the shielding properties of cylindrical shields of arbitrary cross section [1], we introduced several definitions for the time-dependent current and charges on the generalized screen: the local per-unit-length charge density $\rho_L(u^2, z, t)$, the local azimuthal per-unit-length current density $j_{VO}(u^2, z, t)$ and the local axial current on the screen $i_{VA}(u^2, z, t)$. These three real quantities are all dependent on the curvilinear coordinates u^2 and z on the screen and on the time t and can be, respectively, expressed in C/m, A/m, and A.

Manuscript received September 18, 2001; revised July 8, 2002.
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Digital Object Identifier 10.1109/TEMC.2002.804779

Using a Fourier series expansion with respect to the azimuthal coordinate u^2, we then defined the amplitudes $\rho_{Ln}(z,t)$, $j_{VOn}(z,t)$, and $i_{VAn}(z,t)$ of the spectral components, where $n \in \mathbb{N}$. For $n = 0$, they are time-dependent real functions of the coordinate z, and for $n > 0$, they are time-dependent complex functions of the coordinate z.

We then applied a transform consisting of a variation of the Fourier transform on the time variable of these complex functions of time t and z. The result of this transform is a function of the radian frequency ω and of z, into the field of Hamilton's quaternions. We will denote $\boldsymbol{i}$ and $\boldsymbol{j}$ the two real quaternions with a square equal to -1, respectively, corresponding to an angular phase of $\pi/2$ and to a frequential phase of $\pi/2$. In the following, using the same notations for the time-dependent quantities and for the frequency- dependent quantities, amplitudes like $\rho_{Ln}(z,\omega)$, $j_{VOn}(z,\omega)$ and $i_{VAn}(z,\omega)$ will be real quaternions and the $\exp(\boldsymbol{i}nu^2)$ and $\exp(\boldsymbol{j}\omega t)$ dependencies will be omitted.

We showed that the conservation of charges on the cylinder is equivalent to a simple formula ([1, eqs. 17, 18]), relating $\rho_{Ln}(z,\omega)$, $j_{VOn}(z,\omega)$, and $i_{VAn}(z,\omega)$ of same index n.

We introduced the wording tangential response: it was defined along the shield as a distribution of the tangential component of the local current vector and a distribution of the local per-unit-length charge density. Standard responses were defined as tangential responses which can be described using only, for a single value of n, a single amplitude $j_{VOn}(z,\omega)$ or $i_{VAn}(z,\omega)$ or $\rho_{Ln}(z,\omega)$. According to the case, the standard response is said of type j_{VOn}, or i_{VAn} or ρ_{Ln}. Any tangential response of a cylindrical screen, has a unique decomposition into standard responses.

We decided to call "standard excitation at point z" an electromagnetic field distribution created by sources in the volume outside the generalized screen, which would give rise to a locally pure standard response at this point z, if the generalized screen was "ideally metallized," that is to say replaced with a screen having the same external boundary, made perfectly conducting.

In this paper, this theoretical construction will be applied to the characterization of cylindrical generalized screens submitted to the fields of sources in the volume outside the generalized screen. In the case of cylindrical shells, our theory introduces a shielding matrix for the characterization of the screen. In the case of multiconductor shielded cables, this will allow to justify, enhance, and extend our results shown in earlier papers on the five main types of coupling [2], [3].

II. Choice of the External Boundary

Any characterization of the shielding performances of a generalized screen is based on a classification of the possible excitations. For each class of excitation, a "shielding parameter" is then defined, which characterizes the screen's shielding capability. According to our approach, the classification will be based on the use of standard excitations at every point z along the screen. Thus, we will have to define such a shielding parameter for each type of standard excitation. It will be done in Section IV for empty shells and in Section V for multiconductor cables.

Prior to this, we must investigate the relevance of this classification. The classification can be regarded as relevant only if, for any given type of excitation, the response of the shield to any excitation of this type is similar. For instance, as shown in [1, Secs. VII. B, VII. C] for the special case of a circular cylindrical external boundary, there are generally several combinations of impinging cylindrical waves with different angles of incidence (related to the free parameter h), which produce a standard excitation of the same type at a point z.

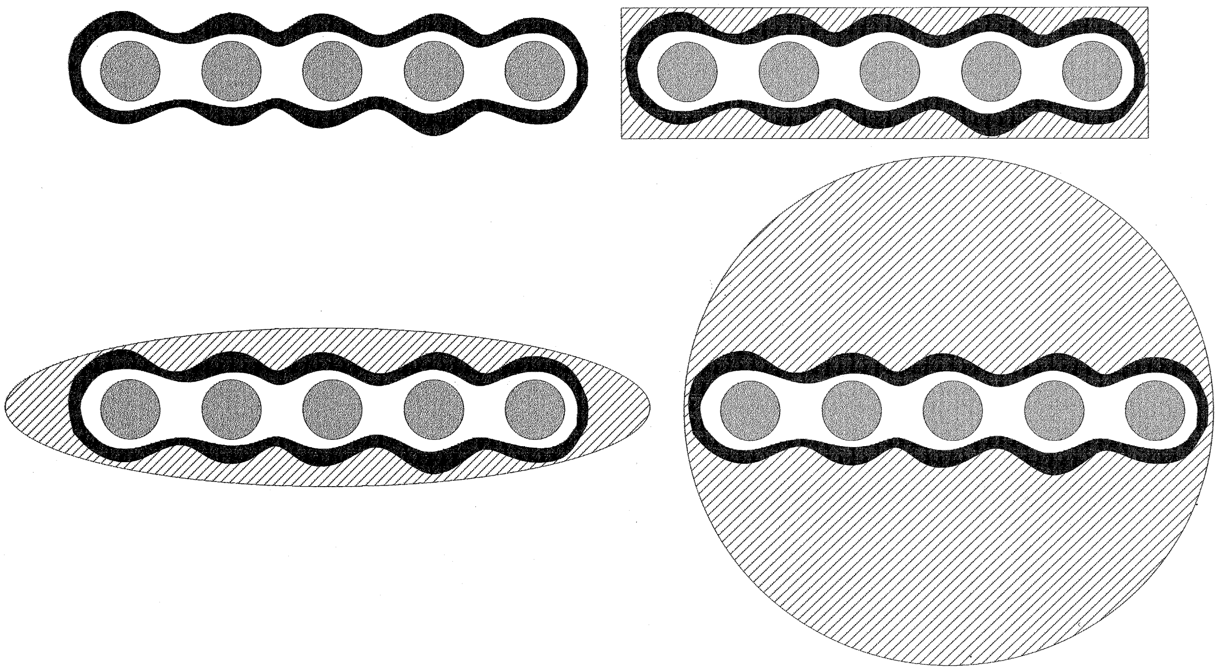

Fig. 1. Flat multiconductor shielded cable: (a) with its screen appearing as a dark area and (b), (c), and (d) with several possible generalized screens containing the cable screen.

If the generalized screen which we wish to characterize had an external boundary with electromagnetic properties close enough from the properties of the ideally metallized generalized screen, any standard excitation of a given type at a point z would, by definition, induce a locally pure standard response of the same type at the point z and, therefore, the same currents and the same charges on the screen. The behavior of the screen at the point z would therefore be the same. Our classification is necessarily relevant for such screens.

However, at this stage, we are not able to establish how similar the currents and charges induced by two different standard excitations of the same type at point z would be, when the generalized screen under consideration has an external boundary with electromagnetic properties differing from those of the ideally metallized generalized screen. This is why we will accept the following conjecture for the implementation of our classification to shields differing from the ideally metallized generalized screen. We assume that any two standard excitations of the same type at point z will give rise to the same behavior of the screen at this point. This conjecture can be viewed as an approximation considered as satisfactory without further examination. The conjecture will, for instance, be useful when we want to characterize a screen having a structure that does not allow charges to flow in any direction on the screen's surface, or when the conducting part of the screen differs from the outer boundary of the generalized screen.

We have also to consider how a characterization can be implemented experimentally and theoretically. Experimentally, for the measurement of the said shielding parameters, a generalized screen under study will be inserted in test setups capable of producing standard excitations of different types. A different test jig will be needed for each outer boundary and each type of standard excitation. Obviously, for economical reasons, few setups will be implemented and only for the simplest generalized screen outer boundaries. On the other hand, the theoretical implementation of the characterization of a generalized screen will typically require, first the computation of incident fields, then, the expansion into a sum of as many standard excitations as needed for the desired accuracy, and, finally, the computation of the amplitude coupled through the shield using the shielding parameters for each of these standard excitations. From the considerations of [1, Secs.VII, VIII, IX], we understand that an expansion into a sum of standard excitations can only be carried out for the simplest shapes of the external boundary of the generalized screen.

Fortunately, the concept of generalized screen is flexible and a first given generalized screen can be combined with an exclusion volume (see [1, Sec. I]) in order to create a second generalized screen with a simpler geometry of the external boundary. If, for instance, we wish to characterize the screen of a flat multiconductor screened cable the cross section of which is shown in Fig. 1 (a), we will not try to compute the standard excitations for a generalized screen having the same boundaries as the cable screen! We may instead consider a generalized screen with a simpler external boundary containing the cable screen, for instance, a generalized screen with a rectangular cylinder as external boundary, as shown in Fig. 1(b), or a generalized screen with a simpler elliptical cylinder as external boundary, as shown in Fig. 1 (c), or a generalized screen with a—simplest—circular cylinder as external boundary, as shown in Fig. 1 (d). In the different drawings of Fig. 1, the hatched areas represent the exclusion volumes and the dark areas represent the screen of the flat cable.

In the remaining parts of this paper, we will therefore only consider the case of a general screen having an outer boundary of revolution. Using the idea of the simplification the outer boundary and our conjecture, the following will nevertheless be applicable to generalized screens with other outer shapes.

III. Natural Amplitude of a Standard Excitation

In order to characterize the shielding performances of a generalized screen, for instance, at an angular frequency ω and at the point z, we will have to compare a set of physical quantities measured in the volume inside the generalized screen, to a value characteristics of the amplitude of the field distribution of a given type of standard excitation, in the neighborhood of $z = 0$. Defining the latter value is the purpose of this paragraph.

The first idea for the definition of the amplitude of a given type of standard excitation, would be to use the amplitude of the electric field or the amplitude of the magnetic field, at a reference point taken in the volume outside the generalized screen, for one of the standard excitation of this type. It would seem natural to use the amplitude of a magnetic field for the standard excitations of type i_{VAn} and j_{VOn} and to use the amplitude of an electric field for the standard excitations of type ρ_{Ln}.

This approach based on a field amplitude at an arbitrary reference point calls for two remarks.

1) It is obvious that by using an arbitrary point in the volume outside the generalized screen, the angle of incidence plays an unwanted role.
2) Also, the more or less arbitrary radius of the generalized screen (see Section II) is likely to play an unnecessary role.

We shall, in the following, in the case of a generalized shield with an outer boundary of circular cross section of radius r_0, define the "natural amplitude" of a standard excitation in a way which will avoid these shortcomings. In order to accomplish this, we shall consider that the amplitude is determined with an ideally metallized screen inserted in the test setup and refer to the electromagnetic field distributions derived at [1, Sec. VII. C], each of them belonging to a given standard excitation at the point $z = 0$ and being defined by a combination of field components. Each field component corresponds to a given value of the propagation constant h, which can take on values in the interval $[-k, k]$, k being the wave number. Thus, there are six cases to be considered in the definition of the natural amplitudes, each of them corresponding to a standard excitation of a given type locally pure in the neighborhood of $z = 0$.

Case 1: For the standard excitation of type i_{VA0}, we shall refer to the combination of a field component $h = k$ and a field component $h = -k$, both of them with the same amplitude c_0. Reference [1, eqs. 30, 32] shows that the azimuthal component H_θ of the magnetic field varies as $1/r$. It therefore sounds appropriate to define the natural amplitude of the standard excitation by the amplitude of the total current i_{VA0} flowing on the ideally metallized screen, which satisfies

$$i_{VA0} = 2\pi r H_\theta \tag{1}$$

for $r \geq r_0$.

Case 2: For the standard excitation of type ρ_{L0} we shall refer to the combination of a field component $h = k$ and a field component $h = -k$, both of them with their amplitudes c_0 of opposite values. [1, eqs. 30, 32] shows that the radial component E_r of the electric field varies as $1/r$. It therefore sounds appropriate to define the natural amplitude of the standard excitation by the amplitude of the total charge ρ_{L0} on the ideally metallized screen, which satisfies

$$\rho_{L0} = 2\pi\varepsilon_0 r E_r \tag{2}$$

for $r \geq r_0$.

Case 3: For the standard excitation of type j_{VO0}, we shall refer to the $h = 0$ component, with all amplitude distributions equal to zero except b_0, and d_0 related by [1, eq. 35]. We shall note that b_0 corresponds to an incident wave, that d_0 corresponds to a reflected wave and that b_0 and d_0 are Dirac distributions. We shall note B_0 the area

$$B_0 = \int_{-k}^{k} b_0(h)dh \tag{3}$$

of the distribution b_0, the unit of B_0 being the amperes meter. We will define the natural amplitude of the standard excitation by the quantity $2k^2 B_0$ on the ideally metallized screen, its unit being amperes per meter. From [1, eqs. 30, 35][4, eq. 9.1.27] we can compute the value of the axial component H_z of the magnetic field for $r \geq r_0$ using Hankel's functions

$$H_z = 2k^2 B_0 \frac{\mathbf{H}_0^{(1)}(kr)\mathbf{H}_1^{(2)}(kr_0) - \mathbf{H}_0^{(2)}(kr)\mathbf{H}_1^{(1)}(kr_0)}{2\mathbf{H}_1^{(2)}(kr_0)}. \tag{4}$$

Which, using [4, 9.1.17] may be simplified into

$$H_z = 2k^2 B_0 \frac{2\boldsymbol{j}}{\pi kr \mathbf{H}_1^{(2)}(kr)}. \tag{5}$$

We note that if the generalized screen has a small radius compared to wavelength, (5) can be transformed using [4, 9.1.9] into

$$H_z = 2k^2 B_0 \frac{\boldsymbol{j}\left(Y_0(kr)J_1(kr_0) - J_0(kr)Y_1(kr_0)\right)}{J_1(kr_0) - \boldsymbol{j}Y_1(kr_0)} \approx 2k^2 B_0 \tag{6}$$

valid for $1 \geq kr \geq kr_0$. When and where these assumptions are valid, the ideally metallized screen is therefore surrounded by a homogenous axial magnetic field. We shall note that, because an infinite solenoidal current sheet of small radius compared to wavelength does not produce any external field (the magnetic field is trapped into the sheet). The field given by (6) is also equal to the field applied by the external sources of the test setup. In the case of a generalized screen of small radius compared to wavelength, the chosen natural amplitude is therefore equal to the magnitude of the homogenous axial magnetic field in the vicinity of the outer boundary, also equal to the applied axial magnetic field.

Case 4: For a standard excitation of type i_{VAn} with $n \geq 1$ we shall refer to the combination of a field component $h = k$ and a field component $h = -k$, with equal amplitudes u_n and v_n, these amplitudes being defined and related respectively by [1, eqs. 33, 36] (the latter containing unfortunately an editorial error). We note that $\delta(h-k)u_n$ is the coefficient of a Φ_n function varying as r^n, caused by sources in the volume outside the generalized screen, whereas $\delta(h-k)v_n$ is the coefficient of a Ψ_n function varying as r^{-n}, caused by current and charges on the ideally metallized screen. Using [1, eq. 30], we find that at $z = 0$ the combination of both terms with the amplitude u_n allows the computation of the quaternion amplitudes of the applied electric field $\mathbf{E}^{\text{appl}}$ and of the applied magnetic field $\mathbf{H}^{\text{appl}}$ taking on the value

$$\begin{cases} E_r^{\text{appl}} = 0 \\ E_\theta^{\text{appl}} = 0 \\ E_z^{\text{appl}} = 0 \\ H_r^{\text{appl}} = 2\boldsymbol{j}\frac{nkr^{n-1}}{\eta_0 r_0^n}u_n\boldsymbol{i} \\ H_\theta^{\text{appl}} = -2\boldsymbol{j}\frac{nkr^{n-1}}{\eta_0 r_0^n}u_n \\ H_z^{\text{appl}} = 0. \end{cases} \tag{7}$$

This formula being valid at $z = 0$, for $r \geq r_0$. We shall define the natural amplitude of the standard excitation by the quantity

$$\frac{H_\theta^{\text{appl}}}{r^{n-1}} = \frac{i_{VAn}}{2\pi r_0^n} \tag{8}$$

where [1, eqs. 34, 36, 37] have been used. In the special case $n = 1$, we note that (7) shows that the applied magnetic field is uniform, orthogonal to the axis and that the natural amplitude of the standard excitation is the amplitude of the applied magnetic field.

Case 5: For a standard excitation of type ρ_{Ln} with $n \geq 1$, we shall refer to the combination of a field component $h = k$ and a field component $h = -k$, with opposite amplitudes u_n and v_n, these amplitudes being related by [1, eq. 36]. As previously, $\delta(h-k)u_n$ is the coefficient of a Φ_n function varying as r^n, caused by sources in the volume outside the generalized screen, whereas $\delta(h-k)v_n$ is the coefficient of a Ψ_n function varying as r^{-n}, caused by current and charges on the ideally metallized screen. Using [1, eq. 30], we find that at $z = 0$ the combination of both terms with the amplitude u_n allows the computation of the applied electric field $\mathbf{E}^{\rm appl}$ and of the applied magnetic field $\mathbf{H}^{\rm appl}$ taking on the value

$$\begin{cases} E_r^{\rm appl} = \mp 2\boldsymbol{j}\frac{nkr^{n-1}}{r_0^n}u_n \\ E_\theta^{\rm appl} = \mp 2\boldsymbol{j}\frac{nkr^{n-1}}{r_0^n}u_n\boldsymbol{i} \\ E_z^{\rm appl} = 0 \\ H_r^{\rm appl} = 0 \\ H_\theta^{\rm appl} = 0 \\ H_z^{\rm appl} = 0. \end{cases} \tag{9}$$

This formula being valid at $z = 0$, for $r \geq r_0$. We shall define the natural amplitude of the standard excitation by the quantity

$$\frac{E_r^{\rm appl}}{r^{n-1}} = \frac{\rho_{Ln}}{2\pi\varepsilon_0 r_0^n} \tag{10}$$

where [1, eqs. 34, 36, 37] have been used. In the special case $n = 1$, we note that (9) shows that the applied electric field is uniform, orthogonal to the axis and that the natural amplitude of the standard excitation is the amplitude of the applied electric field.

Case 6: For standard excitations of type j_{VOn} with $n \geq 1$, electric and magnetic fields are not simple in the neighborhood of $z = 0$. However, we may produce electromagnetic fields of this type by combining a $h = 0$ component, the amplitude distribution a_n, c_n, b_n, and d_n of which are related by [1, eq. 35] and by the additional relation canceling H_θ, with a component $h = k$ and with a component $h = -k$, with opposite amplitudes u_n et v_n related by [1, eq. 36], taking on a value suitable for the cancellation at $z = 0$ of the electric field coordinate E_r of the $h = 0$ component. The latter two components are those of a standard excitation of type ρ_{Ln}. We shall therefore simply define the natural amplitude of the standard excitation of type j_{VOn}, as the natural amplitude of the standard excitation of type ρ_{Ln} so defined.

We have just defined the natural amplitude of all standard excitations, without any reference to an arbitrary point where field values would have been measured. Instead, we have considered that the natural amplitude of the standard excitation is measured with an ideally metallized screen of arbitrary radius installed in the test setup. The definition of the natural amplitude contains no assumption concerning the size of the cross section of the generalized screen with respect to the wavelength.

We can note that in the special case of a screen with a cross section much smaller than the wavelength a vocabulary has already been defined in [2] for configurations referred to as "the five main types of coupling" in [3]. At this point, we can show the relationship between these types of coupling and the types of excitation of this paper:

- a standard excitation of type i_{VA0} corresponds to a Type 1 coupling, the natural amplitude of the standard excitation being the axial current, in agreement with (1);
- a standard excitation of type ρ_{L0} corresponds to a Type 2 coupling, the natural amplitude of the standard excitation being a per-unit-length charge density, in agreement with (2);
- a standard excitation of type j_{VO0} corresponds to a Type 3 coupling, the natural amplitude of the standard excitation being the amplitude of a uniform axial applied magnetic field, in agreement with (6);
- a standard excitation of type i_{VA1} corresponds to a Type 5 coupling, the natural amplitude of the standard excitation being the amplitude of a uniform applied magnetic field parallel to the cross section of the generalized screen, in agreement with (8);
- a standard excitation of type ρ_{L1} corresponds to a Type 4 coupling, the natural amplitude of the standard excitation being the amplitude of a uniform applied electric field parallel to the cross section of the generalized screen, in agreement with (10).

Please note that in the case of the Type 5 coupling, the definitions in [2] and [3] clearly address a standard excitation of type i_{VA1}, whereas the corresponding picture shows a combination of two standard excitations of types i_{VA1} and j_{VO1}.

IV. Hollow Cylindrical Shell

If we want to characterize the shielding properties of a hollow cylindrical shell at a given point z, we will have to compare a set of physical quantities, regarded as effects, measured in the volume inside the generalized screen, to the natural amplitude of a given standard excitation which cause them. In this volume, we know that it is generally possible to compute electric and magnetic fields as if the volume was isolated, provided the effect of external excitations like our standard excitations are taken into account using equivalent sources on its boundary (this boundary is the internal boundary of the generalized screen). It is important to note that these equivalent internal sources are located at the parts of the internal boundary (a surface) where the generalized screen is excited and leaks, whereas the resulting electric and magnetic fields may exists anywhere in the volume inside the generalized screen, even in the case of an excitation limited to a small part of the generalized screen. In addition, equivalent internal sources are to a large extend independent of what could be added in the hollow cylindrical shell, whereas the resulting field are not.

It is therefore appropriate to use the equivalent internal sources, not the resulting fields for the definition of the said physical quantities. In this paper, we will consider that these quantities are the amplitude of source terms related to the currents or charges or electric field or magnetic field amplitudes on the external boundary of the generalized screen, by a linear relation. We will also assume that a finite number of these physical quantities allow a good enough accuracy.

For instance, if we assume that the hollow cylindrical shell has a circular cylindrical internal boundary made of a good homogenous conductor, we can obviously only consider the tangential electric field on the boundary for the equivalent internal sources. If we denote $E_{SA}(\theta, z, t)$ the instantaneous axial component of the tangential electric field and $E_{SO}(\theta, z, t)$ the instantaneous azimuthal component of the tangential electric field, we know that we can expand them using the quaternion peak amplitudes $E_{SAn}(z, \omega)$ and $E_{SOn}(z, \omega)$ in such a way that

$$E_{SA}(\theta, z, t) = \mathrm{Re}\left[\int_0^\infty \mathrm{e}^{\boldsymbol{j}\omega \mathrm{t}}\left\{\sum_{\mathrm{n}=0}^{\infty} \mathrm{E_{SAn}(z}, \omega)\mathrm{e}^{\boldsymbol{i}\mathrm{n}\theta}\right\}\mathrm{d}\omega\right] \tag{11}$$

$$E_{SO}(\theta, z, t) = \mathrm{Re}\left[\int_0^\infty \mathrm{e}^{\boldsymbol{j}\omega \mathrm{t}}\left\{\sum_{\mathrm{n}=0}^{\infty} \mathrm{E_{SOn}(z}, \omega)\mathrm{e}^{\boldsymbol{i}\mathrm{n}\theta}\right\}\mathrm{d}\omega\right]. \tag{12}$$

For a given desired accuracy, higher values of n can be neglected. We have therefore defined a finite set of the wanted physical quantities. We note that if the hollow cylindrical shell had apertures, as in the case of a mesh or braid, it would have been necessary to also consider the instantaneous normal electric field on the boundary $E_{SR}(\theta, z, t)$ and the quaternion peak amplitude components $E_{SRn}(z, \omega)$ of its expansion, defined as

$$E_{SR}(\theta, z, t) = \mathrm{Re}\left[\int_0^\infty \mathrm{e}^{\boldsymbol{j}\omega \mathrm{t}}\left\{\sum_{\mathrm{n}=0}^{\infty} \mathrm{E_{SRn}(z}, \omega)\mathrm{e}^{\boldsymbol{i}\mathrm{n}\theta}\right\}\mathrm{d}\omega\right]. \tag{13}$$

If we take into account such a finite set of physical quantities for the description of the magnitude of the internal equivalent sources, on the one hand and a finite set of natural amplitudes for the measurement of standard excitations of different types on the other hand, the supposedly linear relation between them can be expressed in a matrix con-

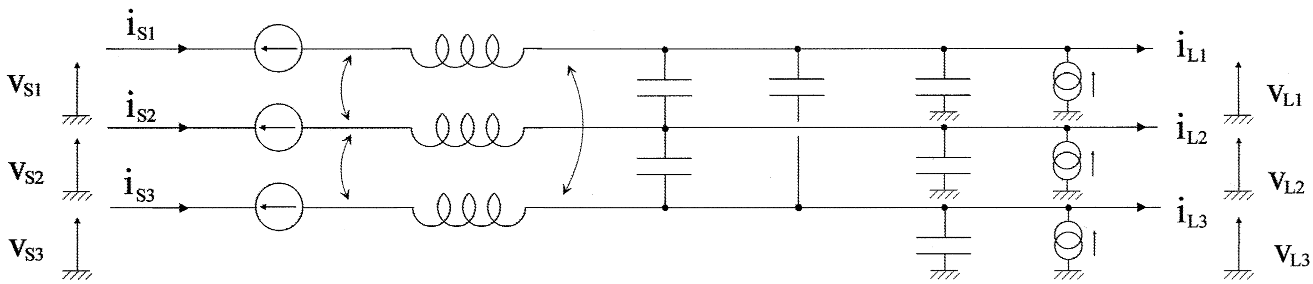

Fig. 2. Short length of shielded multiconductor cable in an external field.

taining the shielding parameters, which we will call "shielding matrix." This shielding matrix would generally depend on the axial coordinate z where the standard excitation is applied and on the axial coordinate z' where the magnitude of the internal equivalent sources is considered.

We will say that the generalized screen is well-behaved if the internal equivalent sources at the point z are practically only related to the incident field values at the same axial coordinate z. In this case, the internal equivalent sources need be considered only where the standard excitation takes place and the shielding matrix depends on a single axial coordinate z. Thin generalized screens, that is to say screens for which the thickness of the hollow cylindrical shell is much smaller than the wavelength of the incident wave, are expected to be well-behaved (for instance, Franceschetti [5] has shown that the homogenous thin shield can be taken into account using only a discontinuous boundary condition, which means that the internal equivalent source depends only on the fields on the opposite side of the screen). Of course, in the case of a well-behaved generalized screen the properties of which do not vary with the axial coordinate z, the shielding matrix does not depend on z either.

The shielding matrix of a well-behaved hollow cylindrical shell is convenient for the characterization of the generalized screen, because it can obviously be used to compute the amplitude of internal equivalent sources and then the internal fields using any appropriate techniques. We note that if the hollow shell is empty, at frequencies below the cutoffof the first propagation mode of this waveguide, internal equivalent sources will only produce evanescent waves, with an essentially local effect. If the frequency is increased above the cutofffrequency, the contribution of the internal equivalent sources will propagate and the fields at any point inside the generalized screen will be the result of the summation of the contributions of the excitations along the whole screen's length and reflections at the termination. If the cylindrical shell contains internal conductors along the axis, the picture is obviously changed because of the presence of a TEM mode available at any frequency for the propagation of local contributions.

V. Shielded Cables

Let us consider a shielded cable with a single shield and N internal conductors, having a cross section much smaller than the wavelength. Only quasi-TEM modes can therefore propagate inside the cable shield. In practice, we are only interested in describing what happens on internal conductors at each end of a section of cable. In a way, the problem of the characterization of the shield is replaced by the problem of the characterization of the cable.

Let us therefore consider a section of cable shorter than the wavelengths of interest. If we first consider the internal problem when no external excitation is applied, we know that, using the shield as a reference conductor, the interaction between the shield and the N internal conductors can be represented using a square matrix of order N as per-unit-length capacitance matrix and a square matrix of order N as per-unit-length inductance matrix. Alternatively, this interaction can be described using an equivalent schematic containing the $N(N+1)/2$ per-unit-length capacitances between the $N+1$ conductors (including N per-unit-length capacitances to the reference conductor) and N per-unit-length self-inductances, one on each internal conductor, with the associated $N(N-1)/2$ per-unit-length mutual inductances. In fact, the per- unit-length capacitance matrix or capacitances describe the effect of the electric field normal to the internal conductors and the per-unit-length inductance matrix or inductances describe the effect of the electric field tangential to the internal conductors.

If we now consider the internal problem when an external excitation is applied, always using the shield as reference conductor, the action of the external sources of the field can be represented using field equivalent sources: per-unit-length current sources for the description of the effect of the electric field normal to the internal conductors of the cable resulting from the external excitation and per-unit-length voltage sources for the description of the effect of the electric field tangential to the internal conductors of the cable resulting from the external excitation. Once redundant sources have been canceled, we obtain an equivalent circuit in which N per-unit-length voltage source and N per-unit-length current sources have been added to the equivalent schematic of the cable without external excitation, as shown on schematic of Fig. 2 for the case $N=3$.

The shielding properties of the shielded cable will be characterized once the complex amplitude of these $2N$ field equivalent sources are established for the relevant different standard excitation, as a function of their natural amplitude. We note that for standard excitations other than the standard excitation of type i_{VA0}, of type ρ_{L0} and of type j_{VO0}, the amplitude of the field equivalent sources is *a priori* dependent on the azimuth angle θ of the applied standard excitation.

Unless the structure of the cable is very weird we can assume that for any low impedance standard excitation (standard excitation of type i_{VAn} and of type j_{VOn}) we can neglect the N per-unit-length current sources and that for a high impedance standard excitation (standard excitation of type ρ_{Ln}) we can neglect the N per-unit-length voltage source. In fact passive circuits exist which can convert a low impedance input into a high impedance output (and vice versa), for instance *LC* circuits at the resonance and transformers, but cable shields are not expected to behave that way!

VI. Five Main Types of Coupling

If we assume that considering only standard excitation of the type i_{VA0}, of the type ρ_{L0}, of the type j_{VO0}, of the type i_{VA1} and of the type ρ_{L1} provides enough accuracy, it is possible to limit the characterization of a shielded cable to the five types of coupling already mentioned at the end of Section III.

In addition, in line with the consideration of the end of Section V on field equivalent sources, we will consider that only Type 1 coupling, Type 2 coupling, and Type 5 coupling have an effect on the amplitude of the N per-unit-length voltage sources and that only Type 2 coupling and Type 4 coupling have an effect on the amplitude of the N per-unit-length current sources. We can now give accurate definitions of the shielding parameters for multiconductor shielded cables.

The Type 1 coupling corresponds to the situation where external sources produce a standard excitation of type i_{VA0} for which the natural amplitude is an axial current. The shielding parameters are therefore N complex per-unit-length transfer impedances (expressed in Ω/m). The Type 1 coupling produces, on a length dz of cable, on the internal conductor α, a voltage dv_α equal to

$$dv_\alpha = Z_{T\alpha} i_{VA0} dz \tag{14}$$

where $Z_{T\alpha}$ is the per-unit-length transfer impedance for the internal conductor α and where i_{VA0} is the applied current flowing on the cable shield.

Type 2 coupling corresponds to the situation where external sources produce a standard excitation of type ρ_{L0} for which the natural amplitude is a per-unit-length charge density. The shielding parameters are therefore N complex frequencies which we prefer to write as the product of $j\omega$ by a dimensionless radial electric coupling coefficient. Type 2 coupling produces, on a length dz of cable, on the internal conductor α, a current di_α equal to

$$di_\alpha = \boldsymbol{j}\omega \zeta_{R\alpha} \rho_{L0} dz \tag{15}$$

where $\zeta_{R\alpha}$ is the radial electric coupling coefficient for the conductor α and where ρ_{L0} is the applied per-unit-length charge density on the cable shield.

Type 3 coupling corresponds to the situation where external sources produce a standard excitation of type j_{VO0} for which the natural amplitude is the amplitude of a uniform axial applied magnetic field. The shielding parameters are therefore N complex transfer impedances (expressed in Ω) which we call axial transfer impedances. Type 3 coupling produces, on a length dz of cable, on the internal conductor α, a voltage dv_α equal to

$$dv_\alpha = Z_{AT\alpha} H_z dz \tag{16}$$

where $Z_{AT\alpha}$ is the axial transfer impedance for the conductor α and where H_z is the amplitude of a uniform axial applied magnetic field.

Type 4 coupling corresponds to the situation where external sources produce a standard excitation of type ρ_{L1}, for which the natural amplitude is the amplitude of a uniform applied electric field parallel to the cross section of the generalized screen. The shielding parameters are therefore N transfer admittances (expressed in S) which we call parallel transfer admittances. They are real quaternions because the azimuth of the applied field is a parameter of the induced current. However, they can be also viewed as complex numbers dependent on the relative azimuth between the applied field and the cable. Type 4 coupling produces, on a length dz of cable, on the internal conductor α, a current di_α equal to

$$di_\alpha = Y_{PT\alpha} E_\perp dz \tag{17}$$

where $Y_{PT\alpha}$ is the parallel transfer admittance for the conductor α, considered as a real quaternion or as an azimuth-dependent complex number and where $E_\perp$ is the amplitude of the uniform applied electric field. We note that this parameter is different from the one defined in [2] and [3] which was later found to be impractical because it involved a charge which could not be easily measured.

Type 5 coupling corresponds to the situation where external sources produce a standard excitation of type i_{VA1} for which the natural amplitude is the amplitude of a uniform applied magnetic field parallel to the cross section of the generalized screen. The shielding parameters are therefore N transfer impedances (expressed in Ω) which we call parallel transfer impedances. They are real quaternions because the azimuth of the applied field is a parameter of the induced current. However, they can be also viewed as complex numbers dependent on the relative azimuth between the applied field and the cable. The Type 5 coupling produces, on a length dz of cable, on the internal conductor α, a voltage dv_α equal to

$$dv_\alpha = Z_{PT\alpha} H_\perp dz \tag{18}$$

where $Z_{PT\alpha}$ is the parallel transfer impedance for the conductor α, considered as a real quaternion or as an azimuth-dependent complex number and where $H_\perp$ is the amplitude of the uniform applied magnetic field.

These definitions of coupling parameters are close to the one given in [2] and [3], with the exception of the one applicable to Type 4 coupling.

VII. Natural Amplitudes for the Cylinder Above a Ground Plane

In the next paragraph we will compute the amplitude of the field equivalent sources in two cases of a cable running parallel to a ground plane. Prior to doing this, we need some more general results on the electrostatic charge distribution on a circular cylinder submitted to an electric field, from which we will determine the values of the natural amplitudes.

We consider an ideal conducting circular cylinder of radius r_0 laying at a height $h - r_0$ above the ideal infinite horizontal ground plane (that is to say, the axis of the cylinder is h above the ground plane). A uniform electrostatic field of intensity E_0 is applied with field lines orthogonal to the ground plane, for instance using a second infinite plane parallel to the ground plane at a height much larger than $r_0 + h$ and connected to a suitable voltage source. We want to determine the charge distribution on the cylinder, using an expansion involving homogenous standard responses ρ_{Ln}, that is to say a standard responses ρ_{Ln} for a charge distribution independent of z.

In order to achieve this, we start by establishing the electrostatic field distributions produced by these standard responses. Let us first consider the field electric field produced on the cylinder boundary by an homogenous (i.e., independent of z) distribution of charges. This field is normal to the surface and its real amplitude $E_L(\theta)$ can be expanded as a angular Fourier series

$$E_L(\theta) = \mathrm{Re}\left[\sum_{n=0}^{\infty} E_{Ln} e^{in\theta}\right]. \tag{19}$$

Using the well-known amplitude ρ_S/ε_0 of the field on the boundary of a conducting cylinder carrying a surface charge density ρ_S and [1, eqs. 4, 5], we obtain that the complex amplitude of the electric field produced by the homogenous standard responses ρ_{Ln} are given by

$$E_{Ln} = \frac{\rho_{Ln}}{2\pi\varepsilon_0 r_0}. \tag{20}$$

Let us now consider the complex potential ζ_0 produced by a cylindrical monopole alone in space placed on the cylinder axis. Taking a point on the cylinder axis as origin and choosing the Ox axis vertical, directed upward, we obtain (see [6, p. 195]) the potential as a function of the cartesian coordinates x and y

$$\zeta_0 = -\frac{p_0}{2\pi\varepsilon_0} \ln(x+iy) \tag{21}$$

where p_0 is the momentum of the cylindrical dipole of order 0, that is to say the per-unit-length charge density. The electric field produced is the opposite of the conjugate of the derivative of the potential and therefore

$$E_x + iE_y = \frac{p_0}{2\pi\varepsilon_0}\frac{1}{x-iy} = \frac{p_0}{2\pi\varepsilon_0}\frac{e^{i\theta}}{r}. \tag{22}$$

Comparing this expression with (20) we of course find that the cylindrical dipole of order 0 produces the same field as an homogenous standard response of type ρ_{L0} with an amplitude $\rho_{L0} = p_0$, because in both cases we have the same field on the boundary $r = r_0$.

For $n \geq 1$, let us consider the complex potential ζ_n' produced by a cylindrical multipole of order n, alone in space at the location of the cylinder axis. We now obtain (see [6, p. 203])

$$\zeta_n' = \frac{p_n}{2\pi\varepsilon_0}\frac{1}{(x+iy)^n} \tag{23}$$

where p_n is the moment of the cylindrical multipole of order n. The resulting field produced is simply

$$E_x + iE_y = \frac{p_n}{2\pi\varepsilon_0}\frac{n}{(x-iy)^{n+1}} = \frac{p_n}{2\pi\varepsilon_0}\frac{ne^{i(n+1)\theta}}{r^{n+1}}. \tag{24}$$

Let us also consider the complex potential ζ_n'' derived from (24) using an analytic inversion (see [7, pp. 220, 250]). We obtain

$$\zeta_n'' = \frac{p_n}{2\pi\varepsilon_0}\frac{(x+iy)^n}{r_0^{2n}}. \tag{25}$$

The field produced by this complex potential is

$$E_x + iE_y = \frac{-p_n}{2\pi\varepsilon_0}\frac{n(x-iy)^{n-1}}{r_0^{2n}} = \frac{-p_n}{2\pi\varepsilon_0}\frac{nr^{n-1}e^{-i(n-1)\theta}}{r_0^{2n}}. \tag{26}$$

If we now define the complex potential ζ_n as $\zeta_n = \zeta_n' - \zeta_n''$, the resulting field at a distance r_0 of the cylinder axis is given by

$$E_x+iE_y = \frac{p_n}{2\pi\varepsilon_0}\frac{ne^{i\theta}}{r_0^{n+1}}\left(e^{in\theta} + e^{-in\theta}\right) = \frac{p_n}{\pi\varepsilon_0}\frac{ne^{i\theta}}{r_0^{n+1}}\cos n\theta. \tag{27}$$

Comparing this result with (20), we find that for $n \geq 1$, the complex potential ζ_n produces the same field as the one which exist when an homogenous standard response of type ρ_{Ln} with an amplitude

$$\rho_{Ln} = \frac{2np_n}{r_0^n} \tag{28}$$

exist on the conducting cylinder, because in both cases we have the same field on the boundary $r = r_0$.

We must now give an interpretation to the two components of the complex potential ζ_n: the complex potential ζ_n' corresponds to the field produced by the charge distribution of the homogenous standard response ρ_{Ln}, whereas the complex potential ζ_n'' corresponds to an external field capable of inducing this charge distribution on the cylinder. Therefore any homogenous charge distribution on the conducting cylinder alone in space, specified using the amplitude of the standard responses ρ_{Ln} produces a complex potential equal to

$$\zeta = -\frac{\rho_{L0}}{2\pi\varepsilon_0}\ln(x+iy) + \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{\infty}\frac{\rho_{Ln}}{n}\frac{r_0^n}{(x+iy)^n}. \tag{29}$$

As a second step, we can now establish the complex potential of the conducting circular cylinder above the ground plane. We must consider a complex potential given in (29), its image complex potential (it would be the opposite of the potential at the opposite of the conjugate point if the origin was on the ground plane) and the complex potential of an homogenous normal field of amplitude E_0 and we therefore obtain

$$\zeta = -\frac{\rho_{L0}}{2\pi\varepsilon_0}\ln\frac{(x+iy)}{-x-iy-2h} + \frac{1}{4\pi\varepsilon_0}\sum_{n=1}^{\infty}\rho_{Ln}\frac{1}{n}\left\{\frac{r_0^n}{(x+iy)^n} - \frac{r_0^n}{(-x-iy-2h)^n}\right\} - (x+iy+h)E_0. \tag{30}$$

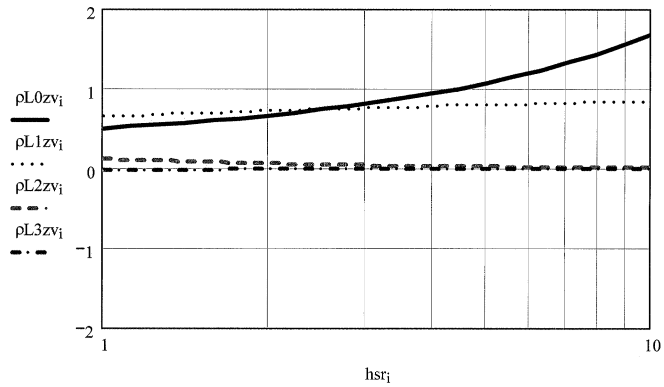

Fig. 3. The dimensionless coefficients ρ_{L0ZV}, ρ_{L1ZV}, ρ_{L2ZV} and ρ_{L3ZV} as a function of the normalized height h/r_0 above a ground plane.

Considering the symmetry of the problem, the amplitudes of the standard responses ρ_{Ln} must be real. At any point on the ground plane, we have $x = -h$ and we can see that, the real part of the complex potential (the real potential) vanishes. The real part of the complex potential must take on a constant value on the boundary of the conducting cylinder. Because of the logarithm, this is only achievable exactly with an infinite number of terms in the series whenever the cylinder is globally charged. If we limit ourselves to an approximate solution with $M+1$ terms, (30) is ideal for the computation of the standard responses using the method of moments with point matching. The system to be solved is

$$\rho_{L0}\ln\left(1+\frac{4h^2}{r_0^2}+\frac{4h}{r_0}\cos\theta\right) + \sum_{n=1}^{\infty}\frac{\rho_{Ln}}{n}\text{Re}\left\{e^{-in\theta} - \frac{(-1)^n}{\left(\frac{2h}{r_0}+e^{i\theta}\right)^n}\right\} = 4\pi\varepsilon_0 E_0 r_0\left(\cos\theta + \frac{h}{r_0}\right) + 4\pi\varepsilon_0 V \tag{31}$$

where V is the real potential of the conducting cylinder with respect to the ground plane, for $M+1$ values θ_m of the azimuth angle θ (the origin of θ is the vertical axis Ox pointing upward), for instance $\theta_m = 2m\pi/(2M+1)$, for m between 0 and M. We have to consider that the general solution is a superposition the solution for $V = 0$ and of the solution for $E_0 = 0$.

For $V = 0$ we have computed (see Fig. 3) the dimensionless

$$\rho_{LnZV} = \frac{\rho_{Ln}}{4\pi\varepsilon_0 r_0 E_0} \tag{32}$$

and for $E_0 = 0$, we have computed (see Fig. 4) the dimensionless

$$\rho_{LnZF} = \frac{\rho_{Ln}h}{4\pi\varepsilon_0 r_0 V} \tag{33}$$

both as functions of h/r_0 ranging between 1 and 10 and for $n = 0$ to $n = 3$. Using the universal plots of Figs. 3 and 4 with the proper value of the normalized height h/r_0, any value of ρ_{L0}, ρ_{L1}, ρ_{L2}, and ρ_{L3} can be computed if one adds the value of ρ_{Ln} given by (32), corresponding to the effect of E_0, to the value of ρ_{Ln} given by (33), corresponding to the effect of V/h. As expected, we can see that there is little difference between the charge distributions for $V = 0$ and $E_0 = 0$ for high values of h/r_0. One should note that one cannot compute ρ_{LnZF} efficiently this way due to a poor convergence when h/r_0 approaches 1. This is because the capacitance between the cylinder and the ground plane (proportional to ρ_{L0ZF}) becomes large and the charge concentrates at $\theta = -\pi$. However, there is a well-known analytic formula for the capacitance between two cylinders (see [7, p. 192]) from which ρ_{L0ZF}

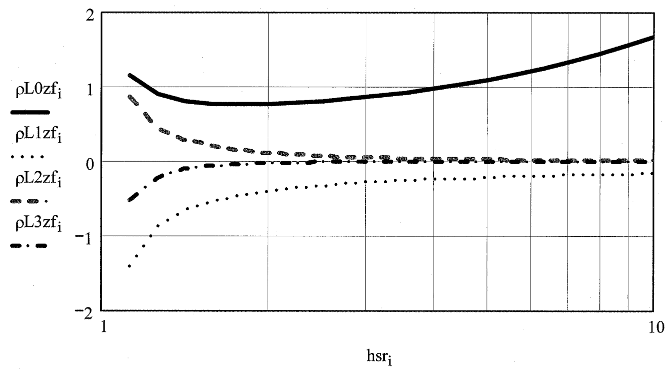

Fig. 4. Dimensionless coefficients ρ_{L0ZF}, ρ_{L1ZF}, ρ_{L2ZF}, and ρ_{L3ZF} as a function of the normalized height h/r_0 above a ground plane.

can be computed for any value of h/r_0. Such a problem does not occur with ρ_{LnZV} and for $h/r_0 = 1$ we obtained

$$\begin{aligned} 2\rho_{L0ZV} &= 1.00\,000 \\ 2\rho_{L1ZV} &= 1.28\,987 \\ 2\rho_{L2ZV} &= 0.22\,849 \\ 2\rho_{L3ZV} &= -0.06\,114 \\ 2\rho_{L4ZV} &= 0.00\,956 \\ 2\rho_{L5ZV} &= 0.00\,278 \\ 2\rho_{L6ZV} &= -0.00\,364 \\ 2\rho_{L7ZV} &= 0.00\,219 \\ \text{and for } n \geq 8 \quad & |2\rho_{LnZV}| \leq 0.001. \end{aligned} \tag{34}$$

This values have been computed for $M = 20$ and for $M = 40$, with the same results for the number of digits shown. We had a few years ago implemented in [2, Appendix I] an expansion with only two terms for an approximate derivation of ρ_{Ln} in the case $h/r_0 = 1$. A "physical" derivation based on the superposition theorem and the image theorem had led us to the approximate values $2\rho_{L0ZV} = 1$ and $2\rho_{L1ZV} = 1$ without numerical computation. In order to obtain this approximate result, one may use [2, eq. 22] noting that the origin of the azimuth angle is a downward pointing axis, with [1, eq. 4] and then (32). This earlier simple derivation was after all not too bad!

Finally, we can now leave the electrostatic field computation and translate our results to the case of an ideally metallized circular cylindrical generalized screen of electrically small cross section. We wish to know the natural amplitudes of the high impedance standard excitations when the axis runs parallel to a ground plane.

For $n = 0$, the natural amplitude can be computed using (2), (32) and (33), using a suitable combination of ρ_{L0ZV} and ρ_{L0ZF}, as

$$\rho_{L0} = 4\pi\varepsilon_0 r_0 \left(E_0 \rho_{L0ZV} + \frac{V}{h} \rho_{L0ZF} \right). \tag{35}$$

The value of V can be computed from the relation between the current injected on the cable (related to ρ_{L0ZV}), the capacitance to ground (related to ρ_{L0ZF}) and the impedances to which the section of generalized screen is connected at each end.

For $n \geq 1$, V being now a known quantity, the natural amplitude defined by (10) can be computed from a suitable combination of ρ_{LnZV} and ρ_{LnZF} as

$$\frac{E_r^{\text{appl}}}{r^{n-1}} = \frac{2}{r_0^{n-1}} \left(E_0 \rho_{LnZV} + \frac{V}{h} \rho_{LnZF} \right) \tag{36}$$

where (32) and (33) have been used.

VIII. Basic Implementation of the Five Main Types of Coupling

We will now establish the expression of the field equivalent sources for two highly interesting situations of field-to-cable coupling in the case of a cable installed above an ideal ground plane: the longitudinal excitation and the transverse excitation for a cable laying on the ground plane. In the case of the longitudinal excitation, a TEM wave propagates along the cable axis. In the case of the transverse excitation, a TEM propagates parallel to the ground plane, orthogonally to the cable axis. The amplitude of the applied field is E_0.

We will consider that the generalized screen of the cable of external radius r_0 is in electrical contact with the ground plane, which implies that $V = 0$ and we will consider only the five main types of coupling. According to (15), (17), (34), (35), and (36), the amplitude of the field equivalent current source for the conductor α will be

$$di_\alpha = (2\boldsymbol{j}\omega\pi\varepsilon_0 r_0 \zeta_{R\alpha} + 1,29 Y_{PT\alpha}) E_0 dz \tag{37}$$

both, in the case of the longitudinal excitation, and transverse excitation.

In the case of the longitudinal excitation, we know that the distribution of charges and current are the same except for a multiplicative constant (see [8, p. 248]) because we are in the case of a TEM wave propagating in an ideal waveguide (the cable shield being in contact with the ground plane, they are viewed as a single conductor by the TEM wave). We can therefore replace E by H and ρ_{L0} by i_{VA0} and ε_0 by 1 in (35) and (36) with $V = 0$. Using (14) and (18), we get the amplitude of the field equivalent voltage source for the conductor α as

$$dv_\alpha = (2\pi r_0 Z_{T\alpha} + 1,29\, Z_{PT\alpha}) \frac{E_0}{\eta_0} dz. \tag{38}$$

In the case of the transverse excitation, the applied axial field can be used directly in (16) and the value of the field equivalent voltage source for the conductor α is therefore

$$dv_\alpha = Z_{AT\alpha} \frac{E_0}{\eta_0} dz. \tag{39}$$

These results can for instance be applied to the case of an electrically short cable for which propagation effects can be disregarded as well as the crosstalk between internal cables. Let us consider that the internal conductor α, is terminated at the near-end with a linear load connected to ground of impedance $Z_{1\alpha}$ and at the far-end with a linear load connected to ground of impedance $Z_{2\alpha}$. We obtain the following values for the near-end induced voltage $v_{1\alpha}$ and the far-end induced voltage $v_{2\alpha}$ in the case of the longitudinal excitation:

$$\begin{aligned} \frac{v_{1\alpha}}{\ell E_0} =& \frac{Z_{1\alpha}}{\eta_0 (Z_{1\alpha} + Z_{2\alpha})} \{1,29 \langle Z_{PT\alpha} \rangle + 2\pi r_0 Z_{T\alpha}\} \\ &+ \frac{Z_{1\alpha} Z_{2\alpha}}{(Z_{1\alpha} + Z_{2\alpha})} \{1,29 \langle Y_{PT\alpha} \rangle + 2\boldsymbol{j}\omega\varepsilon_0 \pi r_0 \zeta_{R\alpha}\} \end{aligned} \tag{40}$$

and

$$\begin{aligned} \frac{v_{2\alpha}}{\ell E_0} =& \frac{-Z_{2\alpha}}{\eta_0 (Z_{1\alpha} + Z_{2\alpha})} \{1,29 \langle Z_{PT\alpha} \rangle + 2\pi r_0 Z_{T\alpha}\} \\ &+ \frac{Z_{1\alpha} Z_{2\alpha}}{(Z_{1\alpha} + Z_{2\alpha})} \{1,29 \langle Y_{PT\alpha} \rangle + 2\boldsymbol{j}\omega\varepsilon_0 \pi r_0 \zeta_{R\alpha}\} \end{aligned} \tag{41}$$

where ℓ is the length of cable submitted to the field, where $\langle Z_{PT\alpha} \rangle$ is the average of $Z_{PT\alpha}$ in the direction of the magnetic field, and where $\langle Y_{PT\alpha} \rangle$ is the average of $Y_{PT\alpha}$ in the direction of the electric field.

In the case of the transverse excitation, we get

$$\frac{v_{1\alpha}}{\ell E_0} = \frac{Z_{1\alpha}}{\eta_0 (Z_{1\alpha} + Z_{2\alpha})} Z_{AT} + \frac{Z_{1\alpha} Z_{2\alpha}}{(Z_{1\alpha} + Z_{2\alpha})} \{1,29 \langle Y_{PT\alpha} \rangle + 2\boldsymbol{j}\omega\varepsilon_0 \pi r_0 \zeta_{R\alpha}\} \tag{42}$$

and

$$\frac{v_{2\alpha}}{\ell E_0} = \frac{-Z_{2\alpha}}{\eta_0 (Z_{1\alpha} + Z_{2\alpha})} Z_{AT} + \frac{Z_{1\alpha} Z_{2\alpha}}{(Z_{1\alpha} + Z_{2\alpha})} \{1,29 \langle Y_{PT\alpha} \rangle + 2\boldsymbol{j}\omega\varepsilon_0 \pi r_0 \zeta_{R\alpha}\}. \tag{43}$$

We note that even though (40)–(43) use quaternions, all terms can be considered as complex numbers, because the azimuth angle θ dependency has been suppressed by the averaging. Measurement results for these voltages, as obtained in a Crawford cell [2] and in a GTEM cell [3] have already been published. The approximate formula for the near-end and far-end voltages in these papers should be replaced with (40)–(43), but their contents remains otherwise valid.

IX. Conclusion

In this paper we have built on basic concepts like the canonical decomposition of the tangential response on a generalized shield and the definition of standard excitations, already introduced in [1], in order to present:

- idea of replacing the exact boundary of the screen with the circular cylindrical boundary of a generalized screen;
- definition of natural amplitudes;
- method for the implementation of these tools for the characterization of cylindrical shells and shielded cable.

This approach of characterization is not perfect and does not solve all problems. When we arbitrarily consider a generalized screen with an outer shape differing a lot from the conducting screen, we might increasingly:

- fail to be able to solve rigorously problems with field sources outside the conducting screen, but falling inside the generalized screen;
- and have to rely more heavily on the speculative aspects of the conjecture of Section II to solve some problems.

We have given new definitions for the five main types of coupling of a shielded cable. These definitions are essentially different from the earlier version: they are not intuitive definitions believed to give an accurate picture of the coupling for a circular cylindrical shield, but the consequence of general basic concepts applicable to both cylindrical shells and shielded cables. We can now see how the five main types of coupling are the first terms of an expansion and understand the underlying assumptions. If case of need, the consequences of limiting one's analysis to the five main types of coupling can now be understood and overcome.

In addition, this paper also contains the computation of the natural amplitudes of the electrical standard excitation for a circular cylindrical generalized screen of electrically small cross section and, for cables, the accurate computation of induced voltages and currents.

References

[1] F. Broydé and E. Clavelier, "The basis of a theory for the shielding by cylindrical generalized screens," *IEEE Trans. Electromagn. Compat.*, vol. 42, pp. 414–426, Nov. 2000.
[2] ——, "Comparison of coupling mechanisms on multiconductor cables," *IEEE Trans. Electromagn. Compat.*, vol. 35, pp. 409–416, Nov. 1993.
[3] ——, "Definition, relevance and measurement of the parallel and axial transfer impedances," in *Proc. 1995 IEEE Int. Symp. EMC*, Atlanta, GA, Aug. 14–18, 1995, pp. 490–495.
[4] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965.
[5] G. Franceschetti, "Fundamentals of steady-state and transient electromagnetic fields in shielding enclosures," *IEEE Trans. Electromagn. Compat.*, vol. EMC-21, pp. 335–347, Nov. 1979.
[6] E. Durand, *Electrostatique—Tome I—Les Distributions*. Paris, France: Masson, 1964.
[7] ——, *Electrostatique—Tome II—Problèmes Généraux—Conducteurs*. Paris, France: Masson, 1966.
[8] R. E. Collin, *Field Theory of Guided Waves*, 2nd ed. Piscataway, NJ: IEEE Press, 1991.
[9] F. Broydé and E. Clavelier, "Correction to "The basis of a theory for the shielding by cylindrical generalized screens"," *IEEE Trans. Electromagn. Compat.*, vol. 44, p. 490, Aug. 2002.