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Two Multiple-Antenna-Port and Multiple-User-Port Antenna Tuners

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***Abstract* — Two multiple-antenna-port and multiple-user-port antenna tuners are studied and compared. The first one is made up of independent and uncoupled single-antenna-port and single-user-port antenna tuners, each having the structure of a π-network. The second one is new. It cannot be separated into independent and uncoupled antenna tuners and it has the structure of a multi-dimensional π-network. It is shown that, unlike fixed matching and/or decoupling networks and unlike the first antenna tuner, the new antenna tuner is able to provide an ideal match at any frequency in a frequency band of intended operation.**

***Index Terms* — Antenna tuning, impedance matching, radio receiver, radio transmitter, MIMO radio communication.**

I. Introduction

We consider a radio device which uses several antennas simultaneously in the same frequency band. The radio device may for instance be a receiver, a transmitter or a transceiver for single-user MIMO radio communication. As shown in Fig. 1, a multiple-antenna-port and multiple-user-port (MAPMUP) antenna tuner is intended to be inserted between the antennas used by the radio device, and the radio device [1]-[4].

A MAPMUP antenna tuner comprises: n antenna ports which may each be coupled to an antenna through a feeder ; m user ports (also referred to as "radio ports") intended to be coupled to the radio device; and p adjustable impedance devices. Here, "adjustable impedance device" refers to any component having two terminals which behave as the terminals of a passive linear two-terminal circuit element, and which present a reactance which is adjustable by mechanical or electrical means.

In a frequency band of intended operation, with respect to the antenna ports and the user ports, the antenna tuner must behave as a passive linear device and its losses should be as low as possible. The antenna ports see an impedance matrix $\mathbf{Z}_{Sant}$ of size $n \times n$ and the user ports present an impedance matrix $\mathbf{Z}_U$ of size $m \times m$. The function of the MAPMUP antenna tuner is to allow an adjustment of $\mathbf{Z}_U$, using a selection of the reactance values of its adjustable impedance devices, so as to obtain or approximate a wanted impedance matrix. In this manner, over a specified frequency band, it is possible to compensate a variation in $\mathbf{Z}_{Sant}$ caused by a change in the frequency of operation, or by the influence of nearby objects and living beings.

Design methods have been proposed for a fixed MAPMUP antenna matching and/or decoupling network, "fixed" meaning that the network is made up of circuit elements having fixed impedances [5]-[7]. These methods are not suitable for MAPMUP antenna tuner design, which must focus on the tuning capability, that is to say the possibility of adjusting $\mathbf{Z}_U$ and compensating variations in $\mathbf{Z}_{Sant}$ [4]. To show the main aspects of MAPMUP antenna tuner design, Section II defines a technical challenge, which is addressed with two different types of MAPMUP antenna tuners, in Section III and Section IV.

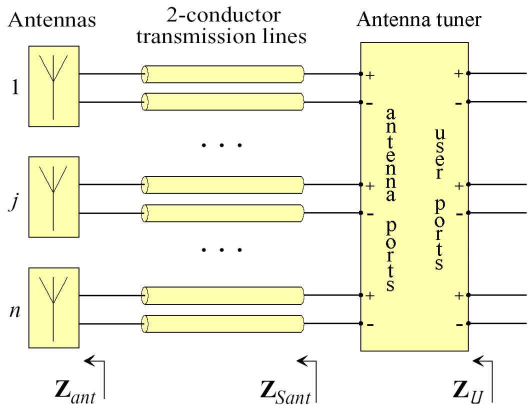

Fig. 1. An array of n antennas coupled to a MAPMUP antenna tuner through n uncoupled 2-conductor transmission lines.

II. A MAPMUP Antenna Tuner Specification

A circular antenna array is made up of n = 4 parallel dipole antennas, each having a total length of 224.8 mm. The radius of the array is 56.2 mm. Each antenna has a 60 mm long feeder. The antenna array is intended to operate in the frequency band 700 MHz to 900 MHz. At the center frequency f_c = 800 MHz, $\mathbf{Z}_{Sant}$ is approximately given by:

$$\mathbf{Z}_{Sant} \approx \begin{pmatrix} 8.6-8.9j & 3.8+4.9j & 1.7+2.2j & 3.8+4.9j \\ 3.8+4.9j & 8.6-8.9j & 3.8+4.9j & 1.7+2.2j \\ 1.7+2.2j & 3.8+4.9j & 8.6-8.9j & 3.8+4.9j \\ 3.8+4.9j & 1.7+2.2j & 3.8+4.9j & 8.6-8.9j \end{pmatrix} \Omega \quad (1)$$

At any frequency, $\mathbf{Z}_{Sant}$ is symmetric and circulant, as shown in (1) at f_c, so that $\mathbf{Z}_{Sant}$ is fully determined by the first three entries of its first row. These entries are plotted in the frequency range 700 MHz to 900 MHz, in Fig. 2.

The technical problem to be solved is the design of an antenna tuner made of ideal (lossless) circuit elements, such that $\mathbf{Z}_U$ can approximate a wanted impedance matrix $\mathbf{Z}_{UW}$, given by

$$\mathbf{Z}_{UW} = r_0 \, \mathbf{1}_4 \quad (2)$$

where r_0 is a resistance and where, for a positive integer q, we use $\mathbf{1}_q$ to denote the identity matrix of size $q \times q$. Here, $n = m = 4$.

Given that the imaginary part of $\mathbf{Z}_{Sant}$ is relatively large (in particular, see curve B of Fig. 2), this problem can obviously not be solved, even approximately, with any fixed MAPMUP antenna matching and/or decoupling network [8].

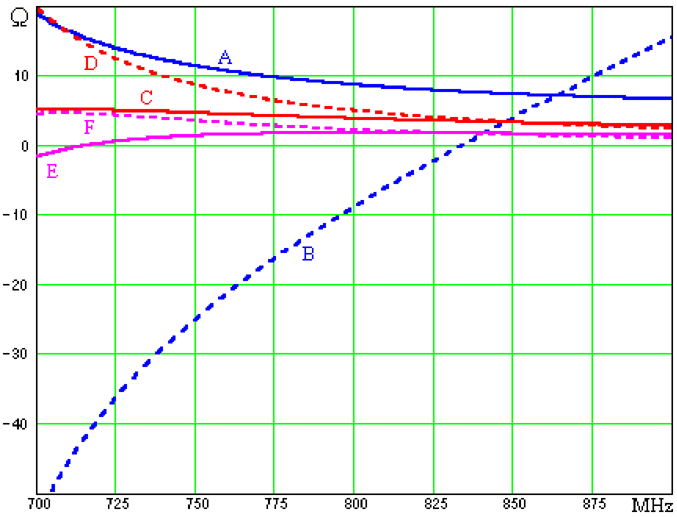

Fig. 2. Entries of $\mathbf{Z}_{Sant}$ versus frequency: Re($\mathbf{Z}_{Sant\ 11}$) is curve A; Im($\mathbf{Z}_{Sant\ 11}$) is curve B; Re($\mathbf{Z}_{Sant\ 12}$) is curve C; Im($\mathbf{Z}_{Sant\ 12}$) is curve D; Re($\mathbf{Z}_{Sant\ 13}$) is curve E; Im($\mathbf{Z}_{Sant\ 13}$) is curve F.

Instead of plotting the entries of $\mathbf{Z}_{Sant}$ and $\mathbf{Z}_U$, we can use the return figure as a figure of merit. In decibels, it is given by [4]

$$F_{dB}(\mathbf{Z}) = 20\log(|||\mathbf{S}(\mathbf{Z})|||_2) \qquad (3)$$

where log is the decimal logarithm, where $|||\mathbf{A}|||_2$ denotes the spectral norm of a matrix $\mathbf{A}$, equal to the largest singular value of $\mathbf{A}$ [9, § 5.6.6 and § 7.3.10], where $\mathbf{Z}$ is an impedance matrix of size $q \times q$, and where $\mathbf{S}(\mathbf{Z})$ is a scattering matrix defined by

$$\mathbf{S}(\mathbf{Z}) = \left(\mathbf{Z} + r_0\, \mathbf{I}_q\right)^{-1}\left(\mathbf{Z} - r_0\, \mathbf{I}_q\right) = \left(\mathbf{Z} - r_0\, \mathbf{I}_q\right)\left(\mathbf{Z} + r_0\, \mathbf{I}_q\right)^{-1} \qquad (4)$$

Let $\mathbf{A}^*$ be the hermitian adjoint of a square matrix $\mathbf{A}$. If $\mathbf{Z}$ is the impedance matrix of a passive device, it is known that $\mathbf{1}_q - \mathbf{S}(\mathbf{Z})\mathbf{S}(\mathbf{Z})^*$ is positive semidefinite, so that by the corollary 7.7.4 of [9], we have $|||\mathbf{S}(\mathbf{Z})|||_2 \leq 1$. Thus, $F_{dB}(\mathbf{Z}_U) \leq 0$dB and $F_{dB}(\mathbf{Z}_{Sant}) \leq 0$dB. In this paper, an ideal match is defined by the condition $\mathbf{Z}_U = r_0\, \mathbf{1}_m$, referred to as "decoupling and matching" by some authors. It corresponds to $F_{dB}(\mathbf{Z}_U) = -\infty$ dB. A possible design target is $F_{dB}(\mathbf{Z}_U) < -10$ dB.

III. A First MAPMUP Antenna Tuner

In this Section III, we study the possibility of addressing the challenge using a MAPMUP antenna tuner made up of $m = n$ independent and uncoupled single-antenna-port and single-user-port (SAPSUP) antenna tuners [10]. More precisely, we only consider the case shown in Fig. 3, where each of the SAPSUP antenna tuners has the structure of a π-network. Let $\mathbf{C}_A$ be the capacitance matrix of the m capacitive adjustable impedance devices coupled to one of the antenna ports, $\mathbf{L}$ the inductance matrix of the m inductive adjustable impedance devices and $\mathbf{C}_U$ the capacitance matrix of the m capacitive adjustable impedance devices coupled to one of the user ports. $\mathbf{C}_A$, $\mathbf{L}$ and $\mathbf{C}_U$ are diagonal matrices. Let ω be the radian frequency. We have

$$\mathbf{Z}_U = \left[\left[\left[\mathbf{Z}_{Sant}^{-1} + j\omega\,\mathbf{C}_A\right]^{-1} + j\omega\,\mathbf{L}\right]^{-1} + j\omega\,\mathbf{C}_U\right]^{-1} \qquad (5)$$

Since the antenna array and the antenna tuner are reciprocal, $\mathbf{Z}_U$ is a complex symmetric matrix which is completely determined by $m\,(m+1)$ real numbers. We use $3m$ adjustable impedance devices each providing a single adjustable real parameter. Unfortunately, for $m \geq 3$, we have $3m < m\,(m+1)$ so that there need not be any possibility of obtaining $\mathbf{Z}_U = \mathbf{Z}_{UW}$. In fact, this analysis applies to any reciprocal MAPMUP antenna tuner made of $n = m \geq 3$ uncoupled SAPSUP antenna tuners [4].

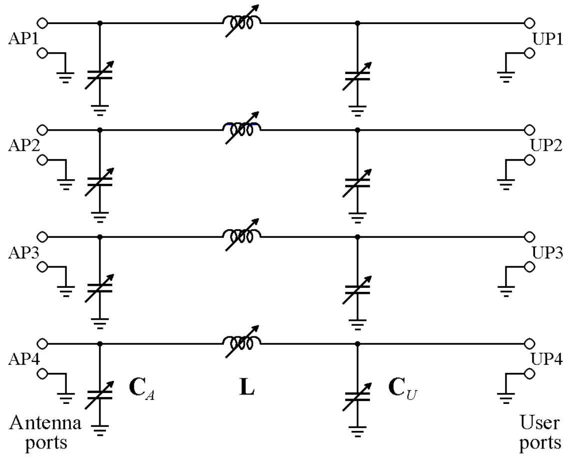

Fig. 3. The MAPMUP antenna tuner considered in Section III, shown in a configuration having $n = 4$ antenna ports, labeled AP1 to AP4, and $m = 4$ user ports, labeled UP1 to UP4.

For the problem defined in Section II, $r_0 = 50\ \Omega$, and at $f_c = 800$ MHz, a numerical analysis leads us to conclude that:

- it is not possible to obtain $\mathbf{Z}_U = r_0\, \mathbf{1}_4$;
- the lowest possible value of $F_{dB}(\mathbf{Z}_U)$ is −4.65 dB, obtained for $\mathbf{C}_A \approx 20.81$ pF $\times\ \mathbf{1}_4$, $\mathbf{L} \approx 2.86$ nH $\times\ \mathbf{1}_4$, and $\mathbf{C}_A \approx 23.75$ pF $\times\ \mathbf{1}_4$, this value of $F_{dB}(\mathbf{Z}_U)$ being indicative of a poor match.

IV. A Second MAPMUP Antenna Tuner

We now investigate the possibility of addressing the challenge using a new MAPMUP antenna tuner which cannot be separated into independent and uncoupled antenna tuners. The schematic diagram of the new antenna tuner is shown in Fig. 4, in the special case $n = m = 4$. This antenna tuner has the structure of a multidimensional π-network. This antenna tuner comprises $n\,(n+1) = m\,(m+1)$ adjustable impedance devices, so that we may hope that there is a possibility of obtaining $\mathbf{Z}_U = \mathbf{Z}_{UW}$ at any frequency in the frequency band of intended operation. The following design achieves this objective.

Let $\mathbf{C}_A$ be the capacitance matrix of the $m\,(m+1)/2$ adjustable impedance devices coupled to one of the antenna ports, $\mathbf{L}$ be the inductance matrix of the windings and $\mathbf{C}_U$ be the capacitance matrix of the $m\,(m+1)/2$ adjustable impedance devices coupled to one of the user ports. $\mathbf{Z}_U$ is again given by (5), in which $\mathbf{C}_A$, $\mathbf{L}$ and $\mathbf{C}_U$ need not be diagonal matrices.

To obtain $\mathbf{Z}_U = r_0\, \mathbf{1}_n$, we need to solve

$$\left(g_0 \mathbf{1}_m - j\omega\,\mathbf{C}_U\right)^{-1} = \left(\mathbf{Z}_{Sant}^{-1} + j\omega\,\mathbf{C}_A\right)^{-1} + j\omega\,\mathbf{L} \qquad (6)$$

where $g_0 = 1/r_0$. Let $\mathbf{G}_A$ and $\mathbf{B}_A$ be real matrices which satisfy $\mathbf{Z}_{Sant}^{-1} = \mathbf{G}_A + j\mathbf{B}_A$. The design of the antenna tuner may for instance be carried out as follows (see Appendix):

step 1 — select an arbitrary physically realizable capacitance matrix $\mathbf{C}_A$ and an arbitrary frequency f_A in the band of interest;

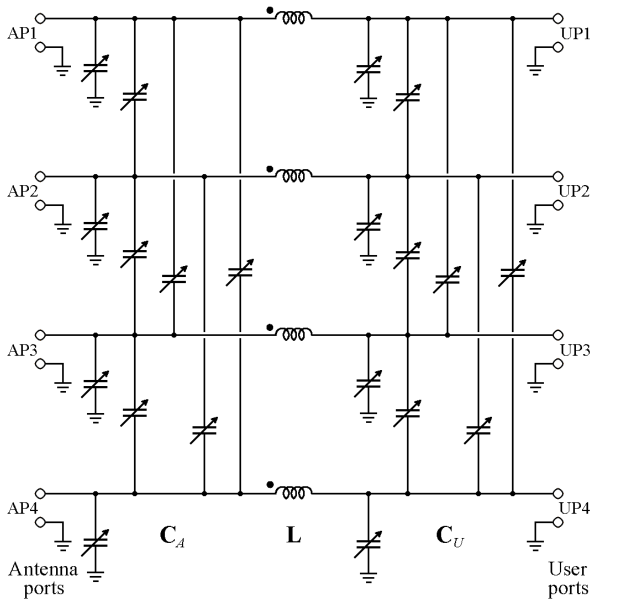

Fig. 4. The new MAPMUP antenna tuner considered in Section IV, shown in a configuration having $n = 4$ antenna ports, labeled AP1 to AP4, and $m = 4$ user ports, labeled UP1 to UP4.

step 2 — compute $\mathbf{C}_U$ compatible with $\mathbf{Z}_U = r_0\, \mathbf{1}_n$ at f_A, using

$$\omega\, \mathbf{C}_U = \left[g_0 \mathbf{G}_A + g_0 \left(\mathbf{B}_A + \omega\, \mathbf{C}_A \right) \mathbf{G}_A^{-1} \left(\mathbf{B}_A + \omega\, \mathbf{C}_A \right) - g_0^2 \mathbf{1}_m \right]^{1/2} \tag{7}$$

where the power 1/2 denotes any square root [11, § 6.4.12], for instance the primary matrix function associated with a suitable choice of square root in $\mathbb{C}$, this principal matrix function being defined for any nonsingular matrix [11, § 6.2.14];

step 3 — compute $\mathbf{L}$ providing $\mathbf{Z}_U = r_0\, \mathbf{1}_n$ at f_A, using

$$\omega\, \mathbf{L} = \left[g_0^2 \mathbf{1}_m + \left(\omega\, \mathbf{C}_U \right)^2 \right]^{-1} \omega\, \mathbf{C}_U + \left[\mathbf{B}_A + \omega\, \mathbf{C}_A + \mathbf{G}_A \left(\mathbf{B}_A + \omega\, \mathbf{C}_A \right)^{-1} \mathbf{G}_A \right]^{-1} \tag{8}$$

step 4 — determine whether $\mathbf{C}_U$ and $\mathbf{L}$ are realizable; if no, go back to step 1; if yes, a physically realizable solution of $\mathbf{Z}_U = r_0\, \mathbf{1}_n$ at f_A has been obtained;

step 5 — $\mathbf{L}$ being fixed, at many frequencies selected in the frequency band of intended operation, determine $\mathbf{C}_U$ and $\mathbf{C}_A$ providing the ideal match $\mathbf{Z}_U = r_0\, \mathbf{1}_n$, using

$$\omega\, \mathbf{C}_A = \left(\omega\, \mathbf{L} \right)^{-1} - \mathbf{B}_A + \mathbf{G}_A \left[\left(g_0 \mathbf{G}_A \right)^{-1} \left(\omega\, \mathbf{L} \right)^{-2} - \mathbf{1}_m \right]^{1/2} \tag{9}$$

first, and then (7);

step 6 — determine whether all $\mathbf{C}_A$ and $\mathbf{C}_U$ obtained at step 5 are realizable; if no, go back to step 1; if yes, a physically realizable antenna tuner has been obtained.

In the case of the problem defined in Section II, for $r_0 = 50\ \Omega$, and using $f_A = f_c = 800$ MHz, we for instance find, at the last iteration caused by the steps 4 and 6:

$$\mathbf{C}_A = \begin{pmatrix} 28.00 & -3.00 & -12.00 & -3.00 \\ -3.00 & 28.00 & -3.00 & -12.00 \\ -12.00 & -3.00 & 28.00 & -3.00 \\ -3.00 & -12.00 & -3.00 & 28.00 \end{pmatrix} \text{pF} \tag{10}$$

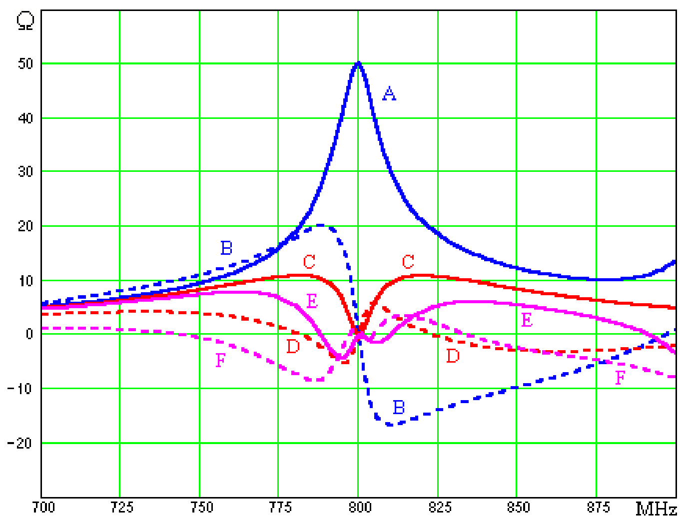

Fig. 5. $\mathbf{Z}_U$ for the initial values of $\mathbf{C}_A$ and $\mathbf{C}_U$: Re($\mathbf{Z}_{U\,11}$) is curve A; Im($\mathbf{Z}_{U\,11}$) is curve B; Re($\mathbf{Z}_{U\,12}$) is curve C; Im($\mathbf{Z}_{U\,12}$) is curve D; Re($\mathbf{Z}_{U\,13}$) is curve E; and Im($\mathbf{Z}_{U\,13}$) is curve F.

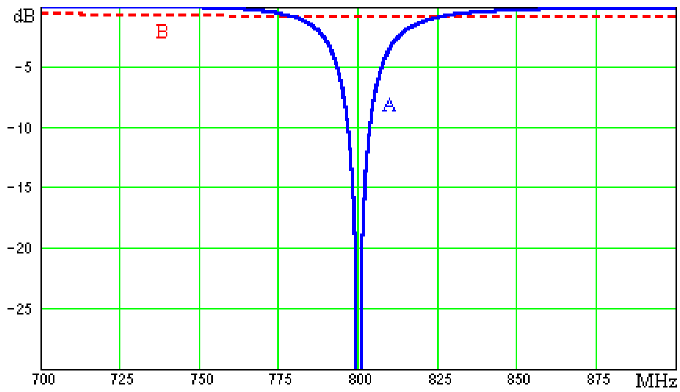

Fig. 6. The return figure versus frequency: F_{dB} ($\mathbf{Z}_U$) for $\mathbf{Z}_U$ shown in Fig. 5 is curve A, and F_{dB} ($\mathbf{Z}_{Sant}$) is curve B.

at the step 1,

$$\mathbf{C}_U \approx \begin{pmatrix} 32.53 & -10.30 & -4.76 & -10.30 \\ -10.30 & 32.53 & -10.30 & -4.76 \\ -4.76 & -10.30 & 32.53 & -10.30 \\ -10.30 & -4.76 & -10.30 & 32.53 \end{pmatrix} \text{pF} \tag{11}$$

at the step 2 and

$$\mathbf{L} \approx \begin{pmatrix} 2.885 & 1.002 & 1.104 & 1.002 \\ 1.002 & 2.885 & 1.002 & 1.104 \\ 1.104 & 1.002 & 2.885 & 1.002 \\ 1.002 & 1.104 & 1.002 & 2.885 \end{pmatrix} \text{nH} \tag{12}$$

at the step 3. The value of $\mathbf{L}$ given by (12) is realizable and can be used to design the windings shown in Fig. 4. The values of $\mathbf{C}_A$ and $\mathbf{C}_U$ given by (10) and (11) are realizable and can be used to design the network of adjustable impedance devices. For $\mathbf{C}_A$, $\mathbf{C}_U$ and $\mathbf{L}$ given by (10)-(12), three entries of $\mathbf{Z}_U$ are plotted in Fig. 5, and F_{dB} ($\mathbf{Z}_U$) and F_{dB} ($\mathbf{Z}_{Sant}$) are plotted in Fig. 6, over the frequency band of intended operation. These plots show that an ideal match $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ is obtained at $f_A = f_c = 800$ MHz.

At the step 5, at each frequency selected in the frequency band of intended operation, we find a new value of $\mathbf{C}_A$ and a new value of $\mathbf{C}_U$ providing the ideal match $\mathbf{Z}_U = r_0\, \mathbf{1}_n$. For instance,

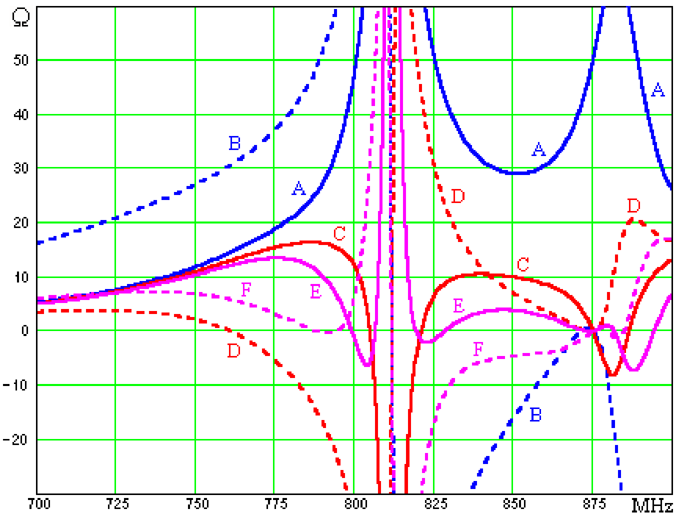

Fig. 7. $\mathbf{Z}_U$ for new values of $\mathbf{C}_A$ and $\mathbf{C}_U$: Re($\mathbf{Z}_{U\,11}$) is curve A; Im($\mathbf{Z}_{U\,11}$) is curve B; Re($\mathbf{Z}_{U\,12}$) is curve C; Im($\mathbf{Z}_{U\,12}$) is curve D; Re($\mathbf{Z}_{U\,13}$) is curve E; and Im($\mathbf{Z}_{U\,13}$) is curve F.

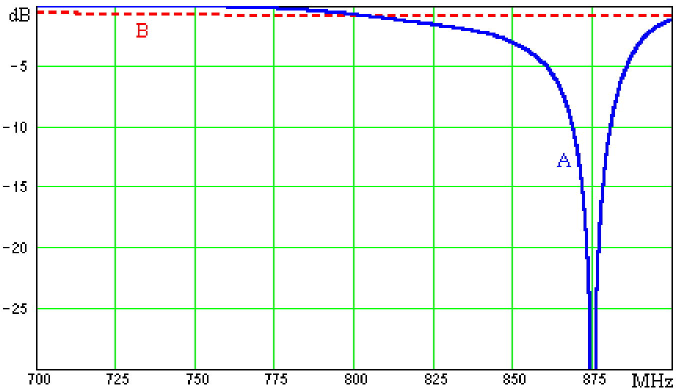

Fig. 8. The return figure versus frequency: F_{dB} ($\mathbf{Z}_U$) for $\mathbf{Z}_U$ shown in Fig. 7 is curve A, and F_{dB} ($\mathbf{Z}_{Sant}$) is curve B.

for the selected frequency 875 MHz, and for the corresponding new values of $\mathbf{C}_A$ and $\mathbf{C}_U$, three entries of $\mathbf{Z}_U$ are plotted in Fig. 7, and F_{dB} ($\mathbf{Z}_U$) and F_{dB} ($\mathbf{Z}_{Sant}$) are plotted in Fig. 8, over the frequency band of intended operation. These plots confirm that an ideal match $\mathbf{Z}_U \approx 50\ \Omega \times \mathbf{1}_4$ is obtained at 875 MHz. In Fig. 6 and Fig. 8, we may observe that the bandwidth for F_{dB} ($\mathbf{Z}_U$) < −10 dB is not very wide.

Each new value of $\mathbf{C}_A$ is symmetric and circulant. Thus, it corresponds to 3 values of the capacitances of the 10 adjustable impedance devices coupled to one of the antenna ports: C_{AG} for the 4 grounded adjustable impedance devices, C_{AN} for 4 others and C_{AF} for the 2 remaining ones. These 3 values are plotted in Fig. 9, versus the selected frequency. In the same way, each new value of $\mathbf{C}_U$ corresponds to 3 values of the capacitances of the 10 adjustable impedance devices coupled to one of the user ports: C_{UG} for the 4 grounded adjustable impedance devices, C_{UN} for 4 others and C_{UF} for the 2 remaining ones. These 3 values are plotted in Fig. 10, versus the selected frequency.

Fig. 9 and Fig. 10 show that the capacitances given by (7) and (9) are real and positive, for our design. Thus, this design is realizable and it can provide an ideal match at any frequency in the frequency range 700 MHz to 900 MHz.

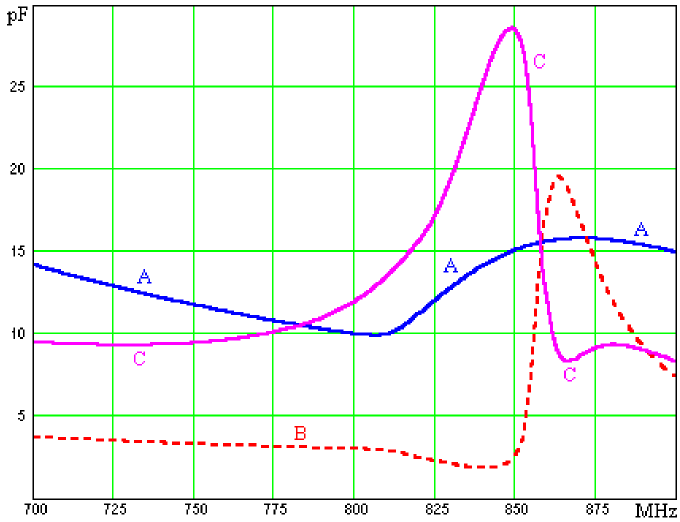

Fig. 9. Capacitances of the adjustable impedance devices which realize $\mathbf{C}_A$: C_{AG} is curve A; C_{AN} is curve B; and C_{AF} is curve C.

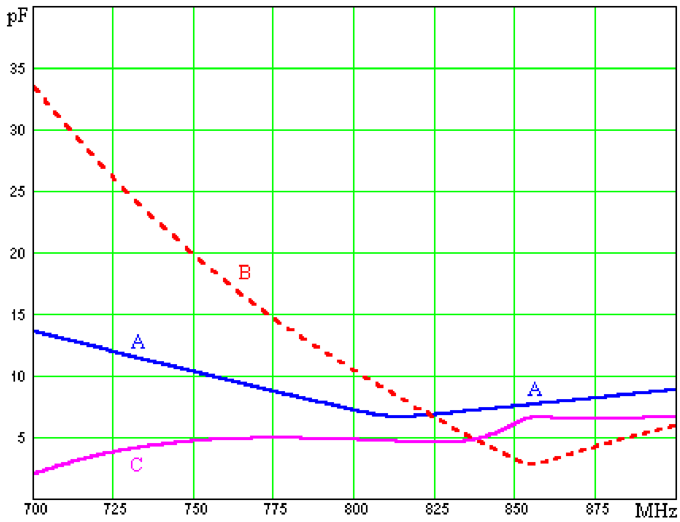

Fig. 10. Capacitances of the adjustable impedance devices which realize $\mathbf{C}_U$: C_{UG} is curve A; C_{UN} is curve B; and C_{UF} is curve C.

V. Conclusion

A MAPMUP antenna tuner made up of n independent and uncoupled SAPSUP antenna tuners, each having the structure of a π-network, was shown to be unable to provide an acceptable approximation of $\mathbf{Z}_U = r_0\,\mathbf{1}_n$, at an arbitrary frequency, for $n \geq 3$ and a nondiagonal $\mathbf{Z}_{Sant}$. Moreover, a fixed MAPMUP matching and/or decoupling network, made up of circuit elements having fixed reactances, can provide an ideal match $\mathbf{Z}_U = r_0\,\mathbf{1}_n$ at a single frequency, but the bandwidth over which an acceptable approximation of this result can be obtained is narrow [8].

In contrast, a new MAPMUP antenna tuner having the structure of a multidimensional π-network is able to provide an ideal match $\mathbf{Z}_U = r_0\,\mathbf{1}_n$, which realizes decoupling and matching, at any frequency in a wide frequency band of intended operation. This is a consequence of the fact that this antenna tuner has a full tuning capability [4]. The new MAPMUP antenna tuner comprises only $n\,(n+2)$ circuit elements, among which $n\,(n+1)$ adjustable impedance devices. It cannot be separated into independent and uncoupled antenna tuners.

A design method has been proposed for the new MAPMUP antenna tuner, in the ideal case of lossless circuit elements. Since it focuses on the tuning capability, the design method is entirely different from the ones proposed for fixed MAPMUP antenna matching and/or decoupling networks. The authors are investigating a generalization of this method, which takes losses into account.

The directivity pattern of the user ports at a given frequency is influenced by the initial choice of $\mathbf{C}_A$ at step 1 [4]. In a portable wireless device such as a mobile phone, for which the orientation is random and time-varying, and for which the directivity pattern is subject to the user interaction, this characteristic should not be a problem, if the MAPMUP antenna tuner can be tuned sufficiently fast.

APPENDIX

In this Appendix, we want to prove (7) to (9). We need a preliminary result on the real and the imaginary parts of the inverse of a square complex matrix. Let **M**, **N**, **P** and **Q** be four square real matrices of size $m \times m$ such that

$$\left(\mathbf{M} + j\mathbf{N}\right)^{-1} = \mathbf{P} + j\mathbf{Q} \tag{13}$$

Assuming that **M** is invertible, it follows that

$$\begin{cases} -\mathbf{Q} = \mathbf{M}^{-1}\mathbf{N}\mathbf{P} \\ \mathbf{M}\mathbf{P} + \mathbf{N}\mathbf{M}^{-1}\mathbf{N}\mathbf{P} = \left(\mathbf{M} + \mathbf{N}\mathbf{M}^{-1}\mathbf{N}\right)\mathbf{P} = \mathbf{1}_m \end{cases} \tag{14}$$

and

$$\begin{cases} \mathbf{P} = \left(\mathbf{M} + \mathbf{N}\mathbf{M}^{-1}\mathbf{N}\right)^{-1} \\ \mathbf{Q} = -\mathbf{M}^{-1}\mathbf{N}\left(\mathbf{M} + \mathbf{N}\mathbf{M}^{-1}\mathbf{N}\right)^{-1} \end{cases} \tag{15}$$

where we assume that $\mathbf{M} + \mathbf{N}\,\mathbf{M}^{-1}\,\mathbf{N}$ is invertible. This is the wanted result. We now want to solve (6). If we use $\mathbf{B}_T$ to denote $\mathbf{B}_T = \mathbf{B}_A + \omega\,\mathbf{C}_A$, the equation to be solved is

$$\left(g_0 \mathbf{1}_m - j\omega\,\mathbf{C}_U\right)^{-1} = \left(\mathbf{G}_A + j\mathbf{B}_T\right)^{-1} + j\omega\,\mathbf{L} \tag{16}$$

Let us first assume that $\mathbf{C}_A$ is known. Applying (15) to both sides of (16), we obtain

$$\begin{aligned} &\left[g_0 \mathbf{1}_m + \frac{1}{g_0}\left(\omega\,\mathbf{C}_U\right)^2\right]^{-1} \\ &\quad = \left[\mathbf{G}_A + \left(\mathbf{B}_A + \omega\,\mathbf{C}_A\right)\mathbf{G}_A^{-1}\left(\mathbf{B}_A + \omega\,\mathbf{C}_A\right)\right]^{-1} \end{aligned} \tag{17}$$

and

$$\begin{aligned} &-\left[\omega\,\mathbf{C}_U + g_0^2\left(\omega\,\mathbf{C}_U\right)^{-1}\right]^{-1} \\ &\quad = -\left[\mathbf{B}_A + \omega\,\mathbf{C}_A + \mathbf{G}_A\left(\mathbf{B}_A + \omega\,\mathbf{C}_A\right)^{-1}\mathbf{G}_A\right]^{-1} + j\omega\,\mathbf{L} \end{aligned} \tag{18}$$

By (17), we have

$$\begin{aligned} \left(\omega\,\mathbf{C}_U\right)^2 &= g_0 \mathbf{G}_A \\ &\quad + g_0\left(\mathbf{B}_A + \omega\,\mathbf{C}_A\right)\mathbf{G}_A^{-1}\left(\mathbf{B}_A + \omega\,\mathbf{C}_A\right) - g_0^2 \mathbf{1}_m \end{aligned} \tag{19}$$

If the right-hand side of (19) has a square root, e.g. because it is nonsingular, (19) can be used to obtain (7) as a possible value of $\mathbf{C}_U$, and (18) can be used to obtain (8) as a possible value of **L**. Let us now consider the problem in which **L** is known but $\mathbf{C}_A$ is unknown. Inverting both sides of (16), we get

$$g_0 \mathbf{1}_m = \left(j\omega\,\mathbf{L} + \left[\mathbf{G}_A + j\mathbf{B}_T\right]^{-1}\right)^{-1} \tag{20}$$

which may be written in the form

$$g_0 \mathbf{1}_m = \left(\mathbf{G}_A + j\mathbf{B}_T\right)\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T + j\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1} \tag{21}$$

By (15), it follows that

$$\begin{aligned} g_0 \mathbf{1}_m &= \left(\mathbf{G}_A + j\mathbf{B}_T\right)\left(\mathbf{1}_m - j\omega\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right)^{-1}\mathbf{L}\mathbf{G}_A\right) \\ &\quad \times \left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T + \omega^2\mathbf{L}\mathbf{G}_A\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right)^{-1}\mathbf{L}\mathbf{G}_A\right)^{-1} \end{aligned} \tag{22}$$

After some manipulations, we obtain

$$\begin{aligned} &g_0\left(\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right)\left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1}\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right) + \omega\,\mathbf{L}\mathbf{G}_A\right) \\ &\quad = \mathbf{G}_A\left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1}\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right) + \mathbf{B}_T \\ &\quad = \left(\omega\,\mathbf{L}\right)^{-1}\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right) + \mathbf{B}_T = \left(\omega\,\mathbf{L}\right)^{-1} \end{aligned} \tag{23}$$

Using $r_0 = 1/g_0$, we get

$$\begin{aligned} &\left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1}\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right)\left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1}\left(\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T\right) \\ &\quad = \left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1} r_0\left(\omega\,\mathbf{L}\right)^{-1} - \mathbf{1}_m \end{aligned} \tag{24}$$

If the right-hand side of (24) has a square root, a possible solution of (24) is

$$\mathbf{1}_m - \omega\,\mathbf{L}\mathbf{B}_T = -\omega\,\mathbf{L}\mathbf{G}_A\left[\left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1} r_0\left(\omega\,\mathbf{L}\right)^{-1} - \mathbf{1}_m\right]^{1/2} \tag{25}$$

for which

$$\mathbf{B}_T = \left(\omega\,\mathbf{L}\right)^{-1} + \mathbf{G}_A\left[\left(\omega\,\mathbf{L}\mathbf{G}_A\right)^{-1} r_0\left(\omega\,\mathbf{L}\right)^{-1} - \mathbf{1}_m\right]^{1/2} \tag{26}$$

By the definition of $\mathbf{B}_T$, we get (9) as a possible value of $\mathbf{C}_A$.

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