Comments and Corrections

Corrections to “Two Reciprocal Power Theorems for Passive Linear Time-Invariant Multiports”

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In [1], we have written several times that $\mathbf{Z}_{PAM}$ is positive semidefinite, where we should have written that $\mathbf{Z}_{PAM}$ has a positive semidefinite hermitian part. We have also written that $\mathbf{Y}_{SAM}$ is positive semidefinite, where we should have written that $\mathbf{Y}_{SAM}$ has a positive semidefinite hermitian part. The 3 following corrections result.

1) A correct wording of Theorem 1 is as follows.

Theorem 1: At any frequency, the parallel-augmented multiport has an impedance matrix, denoted by $\mathbf{Z}_{PAM}$, which depends on $\mathbf{Y}_A$ and has a positive semidefinite hermitian part. Moreover, if the added multiport is reciprocal (i.e., if $\mathbf{Y}_A$ is symmetric) and the original multiport is reciprocal, then $\mathbf{Z}_{PAM}$ is symmetric.

2) In the proof of Theorem 1, the first sentence of the last paragraph should be: “Since all this can be done for any $p \in \{1, \dots, N\}$, we can determine an impedance matrix of the parallel-augmented multiport, which has a positive semidefinite hermitian part because the parallel-augmented multiport is passive.”

3) A correct wording of Theorem 2 is as follows.

Theorem 2: At any frequency, the series-augmented multiport has an admittance matrix, denoted by $\mathbf{Y}_{SAM}$, which depends on $\mathbf{Z}_A$ and has a positive semidefinite hermitian part. Moreover, if the added multiport is reciprocal (i.e., if $\mathbf{Z}_A$ is symmetric) and the original multiport is reciprocal, then $\mathbf{Y}_{SAM}$ is symmetric.

A similar error occurred in the first paragraph of Section V, where, before equation (30), we should have written “We note that $H(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})$ is positive definite, so that, by Lemma 1, $\mathbf{Y}_{S1} + \mathbf{Y}_{S2}$ is invertible”, instead of “We note that $\mathbf{Y}_{S1} + \mathbf{Y}_{S2}$ is hermitian and positive definite, so that it is invertible”.

In the paragraph before Theorem 1 and in the paragraph before Theorem 2, “a new passive LTI multiport” could advantageously be replaced with “a new multiport”.

The denominators of [1, eqs. (31) and (33)] are correct but include one unnecessary level of parenthesis.

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The first words of the proof of Theorem 3 should be “The hermitian part of $\mathbf{Y}_{S2}$ being positive definite”. The last words of the proof of Theorem 3 should be “of Theorem 3”.

The last assertion of Theorem 5 “Moreover, if $\mathbf{Z}_{PAM}$ is symmetric and if $\mathbf{Z}_{PAM21}$, $\mathbf{Z}_{S1}$ and $\mathbf{Z}_{S2}$ are circulant, then $\lambda_{1\max} = \lambda_{2\max}$ and $\lambda_{1\min} = \lambda_{2\min}$” is not correct. The correct assertion is “Moreover, if $\mathbf{Z}_{PAM}$, $\mathbf{Z}_{S1}$ and $\mathbf{Z}_{S2}$ are symmetric and if $\mathbf{Z}_{PAM21}$, $\mathbf{Z}_{S1}$ and $\mathbf{Z}_{S2}$ are circulant, then $\lambda_{1\max} = \lambda_{2\max}$ and $\lambda_{1\min} = \lambda_{2\min}$.”

The first words of the proof of Theorem 5 should be “The hermitian part of $\mathbf{Y}_{S2}$ being positive definite”.

The end of the proof of Theorem 5, from “Since $\mathbf{L}_2$ need not be symmetric”, is not correct, and should be replaced with the following argument.

Using [1, eqs. (34) and (35)] in [1, eqs. (44) and (45)], we get

$$\mathbf{N}_1 = \mathbf{Z}^*_{PAM21}(\mathbf{Y}_{S2} + \mathbf{Y}^*_{S2})\mathbf{Z}_{PAM21} \times(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})(\mathbf{Y}_{S2} + \mathbf{Y}^*_{S2})^{-1}(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})^*, \quad (1)$$

and

$$\mathbf{N}_2 = \mathbf{Z}^*_{PAM12}(\mathbf{Y}_{S1} + \mathbf{Y}^*_{S1})\mathbf{Z}_{PAM12} \times(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})(\mathbf{Y}_{S1} + \mathbf{Y}^*_{S1})^{-1}(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})^*. \quad (2)$$

If $\mathbf{Z}_{PAM}$, $\mathbf{Z}_{S1}$ and $\mathbf{Z}_{S2}$ are symmetric, the transpose of $\mathbf{Z}_{PAM12}$ is $\mathbf{Z}_{PAM21}$ so that the transpose of $\mathbf{N}_2$ is

$$\mathbf{N}_2^T = (\mathbf{Y}_{S1} + \mathbf{Y}_{S2})^*(\mathbf{Y}_{S1} + \mathbf{Y}^*_{S1})^{-1} \times(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})\mathbf{Z}_{PAM21}(\mathbf{Y}_{S1} + \mathbf{Y}^*_{S1})\mathbf{Z}^*_{PAM21}. \quad (3)$$

We need an additional assumption, suitable to allow us to remove: $(\mathbf{Y}_{S2} + \mathbf{Y}^*_{S2})$ and $(\mathbf{Y}_{S2} + \mathbf{Y}^*_{S2})^{-1}$ from (1); and $(\mathbf{Y}_{S1} + \mathbf{Y}^*_{S1})$ and $(\mathbf{Y}_{S1} + \mathbf{Y}^*_{S1})^{-1}$ from (3). A first possibility is that we assume that there exist two complex numbers Z_{S1} and Z_{S2} such that $\mathbf{Z}_{S1} = Z_{S1}\mathbf{1}_m$ and $\mathbf{Z}_{S2} = Z_{S2}\mathbf{1}_n$. A second possibility is that we assume that $\mathbf{Z}_{PAM21}$, $\mathbf{Z}_{S1}$ and $\mathbf{Z}_{S2}$ are circulant, because circulant matrices commute, linear combinations of circulant matrices are circulant, and the inverse of an invertible circulant matrix is circulant [2, Sec. 0.9.6]. Using either assumption, we obtain

$$\mathbf{N}_1 = \mathbf{Z}^*_{PAM21}\mathbf{Z}_{PAM21}(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})^*, \quad (4)$$

and

$$\mathbf{N}_2^T = \mathbf{Z}_{PAM21}(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})^*\mathbf{Z}^*_{PAM21}. \quad (5)$$

By [2, Sec. 1.4.1], the eigenvalues of $\mathbf{N}_2^T$ are the same as those of $\mathbf{M}_2$, counting multiplicity. We then observe that the right hand sides of (4) and (5) are $\mathbf{Z}_{PAM21}^{*}\mathbf{B}$ and $\mathbf{B}\,\mathbf{Z}_{PAM21}^{*}$, respectively, where $\mathbf{B}$ is $\mathbf{Z}_{PAM21}(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})(\mathbf{Y}_{S1} + \mathbf{Y}_{S2})^{*}$. Thus, using [2, Sec. 1.3.22] again, we find that $\mathbf{M}_1$ and $\mathbf{M}_2$ have the same eigenvalues, counting multiplicity, which directly leads to the final assertions of Theorem 5.

We have checked that these modifications have no impact on the rest of the article.

References

[1] F. Broyde and E. Clavelier, "Two reciprocal power theorems for passive linear time-invariant multiports," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 67, no. 1, pp. 86–97, Jan. 2020.

[2] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 2nd ed. New York, NY, USA: Cambridge Univ. Press, 2013.